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Basic concepts in plasma accelerators

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In this article, we present the underlying physics and the present status of high gradient and high-energy plasma accelerators. With the development of compact short pulse high-brightness lasers and electron and positron beams, new areas of studies for laser/particle beam–matter interactions is opening up. A number of methods are being pursued vigorously to achieve ultra-high-acceleration gradients. These include the plasma beat wave accelerator (PBWA) mechanism which uses conventional long pulse ($\sim$100 ps) modest intensity lasers ($I\sim 10^{14}$–10^{16} W cm^{-2}), the laser wakefield accelerator (LWFA) which uses the new breed of compact high-brightness lasers ($<$1 ps) and intensities $>$ 10^{18} W cm^{-2}, self-modulated laser wakefield accelerator (SMLWFA) concept which combines elements of stimulated Raman forward scattering (SRFS) and electron acceleration by nonlinear plasma waves excited by relativistic electron and positron bunches the plasma wakefield accelerator.

In the ultra-high intensity regime, laser/particle beam–plasma interactions are highly nonlinear and relativistic, leading to new phenomenon such as the plasma wakefield excitation for particle acceleration, relativistic self-focusing and guiding of laser beams, high-harmonic generation, acceleration of electrons, positrons, protons and photons. Fields greater than 1 GV cm^{-1} have been generated with monoenergetic particle beams accelerated to about 100 MeV in millimetre distances recorded. Plasma wakefields driven by both electron and positron beams at the Stanford linear accelerator centre (SLAC) facility have accelerated the tail of the beams.

Keywords: plasma; laser; accelerators

1. Introduction

Plasma is an attractive medium for particle acceleration (Dawson 1989) because of the high-electric field it can sustain. Our objective here is to concentrate mainly on the fundamental physics of particle acceleration by relativistic plasma waves that are generated, for example, by intense laser or particle beams. In a plasma-based accelerator (Tajima & Dawson 1979), particles gain energy from a longitudinal plasma wave. To produce relativistic particle beams, the plasma waves have to be sufficiently intense, with a phase speed close to the speed of light c in vacuum.

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A number of laboratory schemes are used to generate intense plasma waves that can accelerate charged particles. The most successful are those based on large amplitude relativistic plasma waves generated by lasers, first proposed by Tajima & Dawson (1979). Particle acceleration by relativistic electron plasma waves has been demonstrated in a number of experiments, the most recent (Faure *et al.* 2004; Geddes *et al.* 2004; Mangles *et al.* 2004) producing more than 100 MeV electron beams in distances of about 1 mm. The resulting accelerating fields as high as 1 GV cm^{-1} have been achieved in these experiments. Note that the maximum accelerating field in high-energy accelerators is of the order of 20 MV m^{-1}, due to cavity breakdown.

Short pulse petawatt or terawatt lasers have now made it possible to study laser plasma interactions at ultra-high intensities or high brightness where the laser–electron interaction becomes highly nonlinear and relativistic resulting in a wide variety of interesting phenomena such as (i) plasma wakefield excitation, (ii) relativistic self-focusing and guiding of lasers in plasma channels, (iii) relativistic self-phase modulation, (iv) photon acceleration, (v) proton acceleration, (vi) harmonic generation, (vii) ultra-high magnetic field generation, etc. Intense lasers are not the only way to produce relativistic waves in plasmas. For a number of years some groups have been using the beat wave process to generate intense relativistic electron plasma waves in plasmas.

The beat wave process relies on using two long laser pulses (≤ 100 ps) of moderate intensity $I \simeq 10^{15}$ W cm^{-1} co-linearly injected into a low-density plasma such that the plasma frequency equals the difference frequency of the two laser beams. Under such conditions, large amplitude relativistic plasma waves are generated with a phase velocity equal to the group velocity of the laser beam which is close to the speed of light.

The generation of relativistic plasma waves using the laser wakefield or the beat wave mechanism was first proposed by Tajima & Dawson (1979) for producing an ultra-high gradient plasma accelerator. But it was not until 1994 that Clayton *et al.* (1993) and Everett *et al.* (1994) demonstrated that high-energy injected electrons could be trapped and accelerated to significant energies by relativistic plasma waves. The main emphasis in the beat wave experiments was to demonstrate particle acceleration by the relativistic plasma wave. In these experiments, the maximum accelerating field gradient E was limited by wave breaking, which occurs for a cold plasma when the plasma wave density perturbation δn equals the mean plasma number density n_0. Wavebreaking occurs when higher harmonics created due to nonlinear processes distort an initially sinusoidal wave into a lopsided triangular waveform with a steep leading edge, the gradient of which eventually becomes infinite. The profile becomes multi-valued and the wave turns over and 'breaks' like an ocean wave on the beach, converting the wave energy into thermal energy of the particle. In reality, the maximum wave amplitude is less than this and is determined mainly by relativistic de-tuning due to the relativistic electron mass increase.

In ultra-high accelerating electric fields, the electron quiver velocity in the plasma wave, defined as $v_{osc} = eE/m_e\omega_p$, is greater than the speed of light, where e is the magnitude of the electron charge, E is the electric field of the plasma wave, m_e is the electron rest mass and ω_p is the electron plasma frequency. The electron quiver velocity in the field of the laser, defined as $v_{osc} = eE/\mathrm{m_e}\omega_L$, in this case E is the laser electric field and ω_L is the laser frequency, for the beat wave case is not relativistic, whereas in the high-brightness laser field it is relativistic. This makes a considerable difference to the physics of nonlinear interactions between the two processes.

Alternatively, instead of laser beams, charged particle beams are also used for generating large amplitude plasma waves (Chen *et al.* 1985). Multi GeV electron and positron beams are used to further enhance the energy of accelerated particles (Katsouleas *et al.* 1999; Lee *et al.* 2001; Joshi *et al.* 2002; Blue *et al.* 2003; Hogan *et al.* 2005) because of the incredibly strong electric fields that are induced in the plasma by space charge effects. Recent results (Hogan *et al.* 2005) demonstrate that multi-GeV energy gain is possible.

Accelerating gradient although important is not the only parameter needed to make a successful accelerator, luminosity and emittance are two others that have to match or better conventional accelerators. For example the next linear colliders being planned will have Luminosity in the region $10^{34}\,\mathrm{cm}^{-2}\,\mathrm{s}^{-1}$ which is beyond the present capability of plasma accelerators. Plasma accelerators are ideal at providing a compact, short pulses accelerator, they may also be useful at increasing the energy of conventional accelerators using the afterburner concept (Joshi & Katsouleas 2003).

2. Relativistic plasma wave acceleration

The strength of the electric field at the focus of high-power short-pulse lasers $E_\perp$ is directly related to the laser intensity by $eE_\perp \approx 30\sqrt{I}$ GeV cm^{-1}. The electric field $E_\perp$ of a laser whose intensity I is $10^{18}\,\mathrm{W\,cm}^{-2}$ is 30 GV cm^{-1} at $10^{21}\,\mathrm{W\,cm}^{-2}$ the field is $\sim$1 TV cm^{-1}. Direct use of the laser field for particle acceleration is not straightforward. Since the electric field of the laser is perpendicular to the propagation direction, the maximum energy gain is limited by the distance the particle moves across the wavefront before the electric field changes sign. However, the situation changes when one uses a plasma into which laser energy can be coupled. Plasma as a medium for particle acceleration has a number of advantages. It has no electrical breakdown limit like conventional accelerating structures which are limited to a maximum field strength of less than 1 MV cm^{-1}. A plasma supports longitudinal plasma waves which oscillate at the plasma frequency $\omega_\mathrm{p} \equiv (4\pi n_0 e^2/m_\mathrm{e})^{1/2}$, where n_0 and m_e are the electron number density and the mass, respectively. In these waves the plasma electrons oscillate back and forth at ω_p irrespective of the wavelength. Therefore, these waves can have arbitrary phase speed, v_ph; relativistic plasma waves have $v_\mathrm{ph} \lesssim c$. The electric field E of relativistic plasma waves with an oscillatory density n_1, can be estimated from Poisson equation, yielding $E = \epsilon\sqrt{n}$ V cm^{-1}, where n is the plasma number density in cm^{-3} and ϵ is the plasma wave amplitude or fractional density bunching n_1/n_0. For a plasma density of $10^{19}\,\mathrm{cm}^{-3}$ accelerating gradients of 1 GV cm^{-1} are possible, which is more than a thousand times larger than in conventional accelerators. This aspect of plasma accelerators is what makes them a very attractive alternative to conventional accelerators. The high-accelerating gradients allows the possibility of building compact 'table top' accelerators rather than multi-kilometre sized present day accelerators. In their seminal paper on plasma-based accelerators, Tajima & Dawson (1979) showed how intense short pulse lasers with a pulse length half the plasma wavelength could generate large amplitude relativistic longitudinal plasma waves. This scheme has become known as the laser wakefield accelerator (LWFA). Alternative schemes to excite plasma waves using large laser pulses are: (i) the plasma beat wave accelerator (PBWA; Tajima & Dawson 1979) where two long laser pulses with a frequency separation equal to ω_p beat together in a plasma to resonantly excite the plasma wave and (ii) the Raman forward scattering (RFS; Mora 2001) instability where one long intense laser pulse is used, this is now called the SMLWFA scheme (Bingham *et al.* 1987, 2004). Alternatively instead of using lasers short relativistic particle beams can also excite large amplitude relativistic longitudinal plasma waves. In a beam-driven plasma wakefield accelerator (PWFA) a large-amplitude relativistic plasma waves is excited by a short, in comparison to the plasma wavelength, high-charge relativistic beam. The Coulomb force of the beam's space charge expels plasma electrons, which rush back in after the beam has passed setting up a plasma oscillation (Chen *et al.* 1985). Both electrons or positrons can be used to excite the plasma wakefield, in the case of positrons electrons from the background plasma are pulled in by the bunch these electrons overshoot and set up the plasma oscillation. Tajima & Dawson (1979) showed that the maximum energy gain ΔW of a particle in a relativistic plasma waves with $v_{ph} \leq c$ is

$$\Delta W = 2\epsilon\gamma^2 m_e c^2, \tag{2.1}$$

where γ is the Lorentz factor associated with the phase velocity of the plasma wave $\gamma = (1 - v_{ph}^2/c^2)^{1/2}$. The phase velocity of the plasma wave is equal to the group velocity v_g of the laser in the plasma, *viz.* $v_{ph} = v_g = c(1-(\omega_p^2/\omega^2)^{1/2} \approx c(1-(\omega_p^2/2\omega^2))$, where ω is the laser frequency; therefore, $\gamma = \omega/\omega_p$ and the maximum energy gain is

$$\Delta W = 2\epsilon \frac{\omega^2}{\omega_p^2} m_e c^2. \tag{2.2}$$

It is clear that for given values of ω there is a trade-off to be considered in choosing ω_p. From the group velocity v_g we see that a low value of ω_p is required to minimize the phase slip of extremely relativistic electrons with respect to the wave while a high value of ω_p is necessary to maximize the accelerating field E. We want to maximize E by increasing ω_p but this minimizes the energy gain ΔW due to phase slip. As the electron accelerates it slips forward in phase and eventually outruns the useful part of the accelerating field. Knowing the plasma wavelength and the velocity difference between the plasma wave and particle $\Delta v \simeq c - v_g$, it is possible to determine the phase slip, $\delta = (\omega_p^2/\omega^2)(Lk_p/2)$, where L is the length of the acceleration stage. Clearly the maximum phase shift cannot exceed π. It can be shown that $5\pi/8$ is a preferable figure, since near zero and π the acceleration is small and inefficient. However, only half this range, i.e. $5\pi/16$ is available. The reason for this is that the plasma is bounded in the traverse direction and consequently there is a radial field in quadrature with the longitudinal field, which produces a strong defocusing force over the first half of the accelerating phase and a strong focusing force over the second half. This limits the acceptable range to only $5\pi/16$. To prevent phase slip the accelerator must be split into stages of length λ_p (ω^2/ω_p^2) $(5/16)$. The maximum energy gain occurs over a distance L $(=\Delta W/eE=2\gamma^2 c/\omega_\mathrm{p})$, which is the limit of the dephasing length.

3. Plasma beat wave accelerator

In the PBWA a relativistic plasma wave is generated by the ponderomotive force of two lasers separated in frequency by the plasma frequency, such that the energy and momentum conservation relations are satisfied, *viz.* $\omega_1-\omega_2=\omega_\mathrm{p}$ and $k_1-k_2=k_\mathrm{p}$, where $(\omega_{1,2}, k_{1,2})$ are the frequencies and wavenumbers of the two lasers, respectively, and k_p is the plasma wave wavenumber.

The beat pattern can be viewed as a series of short light pulses each $\pi c\omega_\mathrm{p}$ long moving through the plasma at the group velocity of light which for $\omega_{1,2}\gg\omega_\mathrm{p}$ is close to c. The plasma electrons feel the periodic ponderomotive force of these pulses. Since this frequency difference matches the natural oscillation frequency of the electron plasma wave, ω_p, the plasma responds resonantly to the ponderomotive force and large amplitude plasma waves would be build up.

If $\omega_\mathrm{p}\ll\omega_{1,2}$ then the phase velocity of the plasma wave $v_\mathrm{ph}=\omega_\mathrm{p}/k_\mathrm{p}=(\omega_1-\omega_2)/(k_1-k_2)=\Delta\omega/\Delta k$ equals the group velocity of the laser beams $v_\mathrm{g}=c(1-\omega_\mathrm{p}^2/\omega_{1,2}^2)^{1/2}$ which is almost equal to c in an underdense plasma. Particles that are injected into the beat wave region with a velocity comparable to the phase velocity of the electron plasma wave, can gain more energy from the longitudinal electric field. Since ω_1 is close to ω_2 and much larger than ω_p, the Lorentz factor γ_p associated with the beat waves is

$$\gamma_\mathrm{p}=\left(1-\frac{v_\mathrm{ph}^2}{c^2}\right)^{-1/2}=\frac{\omega_{1,2}}{\omega_\mathrm{p}}\gg 1. \tag{3.1}$$

The beat wave process is related to stimulated Raman forward scattering (SRFS). Stimulated Raman scattering is the terminology used in plasma physics for the scattering of electromagnetic waves by longitudinal electron plasma waves. If the scattered electromagnetic wave propagates in the same direction as the incident electromagnetic wave we refer to this as forward scattering. Electron plasma waves are also sometimes referred to as Langmuir waves after E. Langmuir, who was the first to discover them. The general equations describing the beat wave and SRFS are similar. It is sufficient to analyse the problem of plasma wave growth and saturation using the relativistic fluid equations for electrons, and Maxwell and Poisson equations.

The equation for the plasma number density perturbation is found to be

$$\left(\frac{\partial^2}{\partial t^2}+\omega_\mathrm{p}^2\right)\delta n_\mathrm{e}=\frac{3}{8}\omega_\mathrm{p}^2\left(\frac{\delta n_\mathrm{e}}{n_0^2}\right)^2\delta n_\mathrm{e}-\frac{n_0}{2}\omega_\mathrm{pe}^2\alpha_1\alpha_2\exp\left(-\mathrm{i}\delta t\right), \tag{3.2}$$

where δ is the density amplitude of the relativistic plasma wave $\alpha_j=eE_\mathrm{j}/m_\mathrm{e}\omega_\mathrm{j}c$, $j=1, 2$, is the normalized quiver velocity in the field of each laser and $\delta=\omega_1-\omega_2$ is the frequency mismatch. (Note that we have dropped the primes on δn_e and $\boldsymbol{E}_\mathrm{j}$.) Rosenbluth & Liu (1972) solved equation (3.2) in the limit of zero pump depletion, i.e. $\alpha_1 = \alpha_2 =$ constant, obtaining

$$\frac{\delta n_{\rm e}(t)}{n_0} = \frac{\delta n_{\rm e}(0)}{n_0} + \frac{1}{4}\alpha_1\alpha_2\omega_{\rm p}t, \tag{3.3}$$

which shows that the plasma wave amplitude grows initially linearly with time. However, due to the second term in the right-hand side of equation (3.3), which is a cubic nonlinearity in $\delta n_{\rm e}$ (cf. the first term in the right-hand side of equation (3.2)) and is due to the relativistic electron mass increase in the field of the Langmuir wave, the amplitude growth will slow down and saturation will eventually occur. Rosenbluth & Liu (1972) showed that the wave saturated at an amplitude level well before reaching the wave breaking limit of $\delta n_{\rm e}/n_0 = 1$. We have

$$\frac{\delta n_{\rm e\ max}}{n_0} = \left(\frac{16}{3}\alpha_1\alpha_2\right)^{1/3} \equiv \epsilon. \tag{3.4}$$

From equation (3.2) we see that the relativistic mass increase of the plasma electrons has the effect of reducing the natural frequency of oscillation. From the continuity equation one finds that the electron quiver velocity in the plasma wave is $v_{\rm osc}/c = \delta n_{\rm e}/n_0$ for $\omega_0/k_0 \simeq c$. The natural frequency of oscillation of the plasma wave is reduced, and is given by

$$\omega_{\rm p}' = \omega_{\rm p}\left(1 - \frac{3}{8}\frac{v_{\rm osc}^2}{c^2}\right)^{1/2} \equiv \omega_{\rm p}\left(1 - \frac{3}{8}\frac{\delta n_{\rm e}}{n_0}\right)^{1/2}. \tag{3.5}$$

It was pointed out by Tang *et al.* (1984) that by deliberately allowing for the relativistic mass variation effect (Sluijter & Montgomery 1965) and having a denser plasma such that the plasma frequency was initially larger than the laser frequency difference the plasma wave would come into resonance as it grew, allowing a larger maximum saturation value to be attained. An increase of about 50% in the saturated wave amplitude can be achieved by this technique.

The longitudinal field amplitude of these relativistic plasma waves can be extremely large with a theoretical maximum obtained from Poisson equation, and it is given by

$$E = \epsilon\sqrt{n_0}\ {\rm V\ cm^{-1}}, \tag{3.6}$$

where ϵ is the Rosenbluth & Liu (1972) saturation value defined by equation (3.4). For the plasma densities of the order of $10^{19}\ {\rm cm}^{-3}$ and saturated values of 30% obtained in present day experiments, the field strength can be of order $10^9\ {\rm V\ cm}^{-1}$, which is close to the Coulomb field of a proton $E_{\rm a}$ at a distance of the order of a Bohr radius a_0, $E_{\rm a} = 5\times10^9\ {\rm V\ cm}^{-1}$. This longitudinal field is capable of producing a GeV electron in a distance of 1 cm.

An important consideration in the beat wave scheme is to have sufficiently intense lasers such that the time to reach saturation is short compared to the ion plasma period. When the time-scale is longer than the latter, the ion dynamics becomes important and the electron plasma wave becomes modulationally unstable by coupling to low-frequency ion density perturbations (Amiranoff *et al.* 1995).

The growth of the plasma wave due to the beat wave mechanism is described by

$$\epsilon = \int_0^t \frac{\alpha_1 \alpha_2 \omega_p}{4} \, dt, \tag{3.7}$$

where $\alpha_{1,2} = eE_{1,2}/m_e\omega_{1,2}c$ is the normalized oscillatory velocity of an electron in the laser fields $E_{1,2}$. As the electron plasma wave grows its electric field amplitude given by equation (2.1) becomes large enough that the velocity of an electron oscillating in this field becomes relativistic and the plasma frequency ω_p suffers a small red shift $\Delta\omega_p = -(3/16)\epsilon^2$ due to the relativistic increase in the electron mass. This red shift in the frequency causes the wave to saturate at

$$\epsilon_{sat} = \left(\frac{16}{3}\alpha_1\alpha_2\right)^{1/3}, \tag{3.8}$$

and the time for saturation is

$$\tau_{sat} = \frac{8}{\omega_p}\left(\frac{2}{3}\right)^{1/3}\left(\frac{1}{\alpha_1\alpha_2}\right)^{2/3}. \tag{3.9}$$

Other factors which can limit the interaction or acceleration length is diffraction of the laser beams or the pump depletion. Diffraction limits the depth of focus to the Rayleigh length which may be overcome by channelling of the laser (Milchberg *et al.* 2005). The pump depletion can be avoided by using more powerful lasers. By using intense short pulse lasers ion instabilities, such as the stimulated Brillouin and plasma modulational instabilities, can be avoided. A number of experiments have been carried out which demonstrate that the theoretical estimates are in very good agreement with observations.

The experiments carried out at UCLA (Clayton *et al.* 1993; Everett *et al.* 1994) focused a two frequency carbon dioxide laser and injected a 2 MeV electron beam to the same point in a hydrogen plasma at a density of about 10^{16} cm^{-3}. The results showed that approximately 1% or 10^5 electrons of the randomly phased injected electrons are accelerated from 2 to 30 MeV in the diffraction length of about 1 cm. This corresponds to a gradient of 0.03 GV cm^{-1}. The measured amplitude of the relativistic plasma waves is 30% of its cold wavebreaking limit, agreeing with the theoretical limit given by equation (3.8). What is particularly significant about this experiment is that it demonstrated that the electrons were 'trapped' by the wave potential. Only trapped electrons can gain the theoretical maximum amount of energy limited by de-phasing which occurs when the polarity of the electric field of the plasma wave seen by the accelerated electron changes sign.

A trapped electron, by definition, moves synchronously with the wave at the point of reflection in the wave potential. At this point, trapped electrons have a relativistic Lorentz factor $\gamma = (1 - v_{ph}^2/c^2)^{1/2}$. As the electron continues to gain energy it remains trapped (and eventually executes a closed orbit in the wave potential). Trapping also bunches electrons. In the UCLA experiment (Clayton *et al.* 1993), the plasma wave has a Lorentz factor of 33 which is synchronous with 16 MeV electrons. Therefore, all electrons observed above 16 MeV are trapped and move forward in the frame of the wave.

The experiment done at the Ecole Polytechnique (Amiranoff *et al.* 1995) also accelerated electrons but were limited to very small energy gains from 3 to 3.7 MeV due to the saturation of the relativistic plasma wave by the modulational instability of the plasma wave coupling to the low frequency in acoustic mode. This instability is important for long pulses of the order of the ion plasma period $\omega_{\rm pi}^{-1}$, and it limits the wave amplitude to very small values. All beat wave experiments confirm earlier simulations (Mori *et al.* 1988) and theoretical work demonstrated the need to use short pulses to avoid competing instabilities.

The success of the experiments indicate that it should be possible to accelerate electrons to 1 GeV in a single stage laser plasma beat wave accelerator. In such an experiment an injected 10 MeV beam of electrons of 100 A could produce about 10^8 electrons at 1 GeV energies. The necessary laser power required is ~14 TW (14×10^{12} W) with a pulse duration of 2 ps, corresponding to laser energy of 28 J and wavelengths of 1.05 and 1.06 μm in a plasma with density 10^{17} cm^{-3}. From these parameters we find that the plasma wave will saturate at a value of $\delta n/n_0 \simeq 0.45$, resulting in a field gradient $E = 0.45\sqrt{n_0} \simeq 140$ MV cm^{-1} for $n_0 \simeq 10^{17}$ cm^{-3}. Assuming no self-focusing or laser guiding this accelerating field is constant over a Rayleigh length $R \simeq \pi\theta^2/\lambda \simeq 0.34$ cm, where θ is the spot size. This results in a maximum energy gain of

$$\Delta W = eE\pi R \simeq 150\,\text{MeV}. \tag{3.10}$$

By using a discharge channel or capillary channelling is possible which could increase the interaction length considerably, for an interaction length of about 3 cm a GeV is possible.

One of the problems to overcome is the ion instabilities due to the small ion plasma period, which is of order 15 ps. To avoid ion plasma instabilities, such as the ion modulational instability, the plasma density could be reduced. This has the effect of making the laser beams appear shorter, but it also reduces the maximum accelerating gradient.

4. Laser wakefield accelerator

The LWFA scheme is the most successful to date with three research groups reporting spectacular results on the formation of monoenergetic electron beams with energies in the range 100 MeV (Faure *et al.* 2004; Geddes *et al.* 2004; Mangles *et al.* 2004). Many experiments have been carried out to demonstrate the excitation of the plasma wakefield, but the three experiments at RAL, LBL and LOA (Faure *et al.* 2004; Geddes *et al.* 2004; Mangles *et al.* 2004) were the first to demonstrate monoenergetic beams. The experiments used similar 10–30 TW of laser power focused into a 2 mm long gas jet with a density of about 2×10^{19} cm^{-3}. Monoenergetic electron distribution functions λ observed with energy spreads ranging from about 2 to 24% and energies in the range 80–170 MeV. The beams contained a few times 10^9 electrons. The angular spread of the beams is comparable to conventional radio-frequency systems. The pulse length of the beams are about 10 fs which makes them extremely bright sources for ultrafast time resolved studies.

In the LWFA scheme, a short laser pulse, whose frequency is much larger than the plasma frequency, creates a blowout region or bubble (Joshi & Mori 2005; Pukhov & Gordienko 2005; at $\omega_{\rm p}$) due to the ponderomotive force. Since the plasma wave is not resonantly driven, as in the beat wave, the plasma density does not have to be of a high uniformity to produce large amplitude waves. As an intense pulse propagates through an underdense plasma, $\omega_0 \gg \omega_p$, where ω_0 is the laser frequency, the ponderomotive force associated with the laser envelope $F_{\text{pond}} \simeq -(m/2)\nabla v_{\text{osc}}^2$ expels or blows out electrons from the region of the laser pulse and excites highly nonlinear, relativistic electron plasma waves. Intense nonlinear plasma waves are generated as a result of being displaced by the leading edge of the laser pulse. If the laser pulse length ($c\tau_L$) is long compared to the electron plasma wavelength, then the energy in the plasma wave is re-absorbed by the trailing part of the laser pulse. However, if the pulse length is approximately equal to or shorter than the plasma wavelength, *viz.* $c\tau_L \simeq \lambda_p$, the ponderomotive force excites a wakefield with a phase velocity equal to the laser group velocity, and is not re-absorbed. Thus, any pulse with a sharp rise or a sharp fall on a scale of c/ω_p will excite a wake. With the development of high-brightness lasers the laser wakefield concept (Tajima & Dawson 1979) has now become a reality. The focal intensities of such lasers are $\geq 10^{19}$ W cm^{-1}, with $v_{\text{osc}} \geq 1$, which is the strong nonlinear relativistic regime. Any analysis must, therefore, be in the strong nonlinear relativistic regime and a perturbation procedure is invalid.

The scaling of the electric field amplitude is more complicated than for the beat wave, because of the strong nonlinear structure known as the bubble. The maximum wake electric field amplitude generated by a plane polarized laser pulse has been given by Sprangle *et al.* (1988) in the one-dimensional limit as $E_{\max} = 0.38(v_{\text{osc}}/c)^2(1 + v_{\text{osc}}^2/2c^2)^{-1/2}\sqrt{n_0}$ V cm^{-1} for $v_{\text{osc}}/c \sim 4$, and $n_0 = 10^{18}$ cm^{-1}, $E_{\max} \approx 2$ GV cm^{-1}, and the time to reach this amplitude level is of the order of the laser pulse length. There is no growth phase as in the beat wave situation, which requires many plasma periods to reach its maximum amplitude. Other estimates have been given by Joshi & Mori (2005) and Pukhov & Gordienko (2005). The experiment also measured laser pulse distortion as a result of the nonlinear interaction between the laser and plasma in setting up the ultra strong relativistic plasma wave (Mori *et al.* 1988). Simulations (Joshi & Mori 2005) have demonstrated the feasibility of producing TeV electrons in a plasma channel using this scheme. The plasma length for the simulation is 200 m.

There is a significant distortion of the laser pulse resulting in photon spikes. The distortion occurs where the wake potential has a minimum and the density has a maximum. The spike arises as a result of the photons interacting with the plasma density inhomogeneity with some photons being accelerated (decelerated) as they propagate down (up) the density gradient; this effect, predicted again by John Dawson and his group (Wilks *et al.* 1992) and by others (Mendonça 2001), is called the photon accelerator. The distortion of the trailing edge increases with increasing ω_{p0}/ω_0. The longitudinal potential, *viz.* $e\phi/mc^2 > 1$ or $eE_z/m_e\omega_{p0}c > 1$, is significantly larger than fields obtained in the plasma beat-wave accelerator, which are limited by relativistic detuning, no such saturation exists in the LWFA. Furthermore, Reitsma *et al.* (2002) proposed a new regime of laser wakefield acceleration of an injected electron bunch with strong bunch wakefields. In particular, the transverse bunch wakefield induces a strong self-focusing that reduces the transverse emittance growth arising from misalignment errors.

5. Self-modulated laser wakefield accelerator

SMLWFA is a hybrid scheme combining elements of SRFS (Mora 2001) and the laser wakefield concept. RFS describes the decay of a light wave at frequency ω_0 into light waves at frequency $\omega_0 \pm \omega_p$, and a plasma wave ω_p with $v_{ph} \simeq c$. Although RFS generates relativistic plasma waves and was identified as the instability which generated MeV electrons in early laser plasma experiments, it was not considered a serious accelerator concept because the growth rate is too small for sufficient plasma wave amplitudes to be reached before the ion dynamics disrupt the process. However, coupled with the LWFA concept it becomes a viable contender. Short pulse lasers have been demonstrated in Mori *et al.* (1994) and Andreev *et al.* (1995) to self-modulate in a few Rayleigh lengths. This modulation forms a train of pulses with approximately $\pi c/\omega_p$ separation, which act as individual short pulses to drive the plasma wave. The process acts in a manner similar to a train of individual laser pulses. A comprehensive theoretical and simulation studies of RFS and self-modulation pulses in tapered plasma channels have been presented by Penano *et al.* (2002).

A number of groups (e.g. Modena *et al.* 1995; Nakajima *et al.* 1995; Wagner *et al.* 1997; Gordon *et al.* 1998; Santala *et al.* 2001) have recently reported experimental evidence for the acceleration of electrons by relativistic plasma waves generated by a modulated laser pulse. The most impressive results come from a group working with the Vulcan laser at RAL, UK; this group, which consisted of research teams from Imperial College, UCLA, Ecole Polytechnique and RAL, has reported (Gordon *et al.* 1998; Santala *et al.* 2001) observations of electrons at energies as high as 120 MeV. The observations of energetic electrons was correlated with the simultaneous observation of $\omega_0 + n\omega_p$ radiation generated by RFS. The experiments were carried out using a 25 TW laser with intensities less than 10^{18} W cm^{-2} and pulse lengths less than 1 ps in an underdense plasma $n_0 \sim 10^{19}$cm^{-3}. The laser spectrum is strongly modulated by the interaction, showing sidebands at the plasma frequency. Electrons with energies up to 100 MeV with an inferred minimum acceleration gradient of >1.60 GV cm^{-1} from >100 MeV electrons over a measured 600 μm interaction length have been observed (Gordon *et al.* 1998). Laser self-channelling of up to 12 Rayleigh lengths was also observed in the experiment. Such extensive self-channelling was only observed for electron energies up to 40 MeV; at the higher energy ~ 94 MeV the acceleration length was 7 times shorter. Similar results of a modulated laser pulse at ω_p have been obtained by a separate Livermore experiment (Coverdale *et al.* 1995) using a 5 TW laser but only observed 2 MeV electrons. The difference in the energy is explained by a difference in the length of plasma over which acceleration takes place. In the RAL experiment the acceleration length is much larger. It is worth pointing out that in these experiments the accelerated electrons are not injected but are accelerated out of the background plasma. These experiments together with the short pulse wakefield have produced the largest—terrestrial acceleration gradients ever achieved—almost 200 GeV m^{-1}.

The RFS instability is the decay of an electromagnetic wave (ω_0, k_0) into a forward propagating plasma wave (ω_p, k_p) and forward propagating electromagnetic waves the Stokes wave at $(\omega_0, -n\omega_p)$ and an anti-Stokes wave at $(\omega_0, +n\omega_p)$, where n is an integer; this is a four wave process. One of the earliest papers on forward Raman instabilities in connection with laser plasma accelerators was by Bingham (1983) and McKinstrie & Bingham (1992) who discussed a purely temporal theory including a frequency mismatch. More recently a spatial temporal theory was developed (Mori *et al.* 1994; Decker *et al.* 1996). In this theory the relativistic plasma wave grows from noise; calculating the noise level is non-trivial since there are various mechanisms responsible for generating the noise. For example, the fastest growing Raman backscatter and sidescatter instabilities cause local pump depletion forming a 'notch' on the pulse envelope. The plasma wave associated with this notch acts as an effective noise source as seen in the simulations by Tzeng *et al.* (1996). Simulations carried out by Tzeng *et al.* (1996) are the first to use the exact experimental parameters and show significant growth within a Rayleigh length.

All these experiments rely heavily on extending the acceleration length which is normally limited to the diffraction length or Rayleigh length $L_R = (\omega_0/2c)\sigma_0^2$, where ω_0 is the laser frequency and σ_0 is the spot size. In present day experiments this is limited to a few mm, for example the RAL Vulcan CPA laser has a wavelength of 1 μm, a spot size of 20 μm resulting in a Rayleigh length $L_R \simeq 350$ μm. To be a useful accelerator laser pulses must propagate relatively stably through uniform plasmas over distances much larger than the Rayleigh length. Relativistic self-focusing is possible if the laser power exceeds the critical power given by $P_c \approx 17\omega_0^2/\omega_p^2$ GW, which is easily satisfied for the present high-power laser experiments. There are difficulties in relying on the laser pulse to form its own channel since the beam may break up due to various laser-plasma instabilities, such as Raman scattering and filamentation. Relativistic self-guiding over five Rayleigh lengths has recently been reported by (Chiron *et al.* 1996) using a 10 TW laser in a plasma of density 5×10^{18} cm^{-3}. They also note that the effect disappears at larger powers and densities. Alternatively, plasma channels have been demonstrated (Durfee *et al.* 1995) to be very effective in channelling intense laser pulses over distances much larger than the Rayleigh length. In these experiments, a two laser pulse technique is used. The first pulse creates a breakdown spark in a gas target, and the expansion of the resulting hot plasma forms a channel which guides a second pulse injected into the channel. Pulses have been channelled up to 70 Rayleigh lengths (Durfee *et al.* 1995) corresponding to 2.2 cm in the particular experiment with about 75% of the energy in the injected pulse focal spot coupled into the guide.

6. Particle beam driven plasma wakefield accelerators

The laser accelerator schemes are very effective at producing very high-accelerating gradients over short plasma scales of the order of mms. Channelling coupled with guiding can increase this to several centimetres producing GeV's on a table top. Extending these table top accelerators needed for high-energy physics requires meter scale plasma, alternatively high energies could be accomplished by staging hundreds of these relatively compact laser accelerators, each providing gains of up to 10 GeV. However, staging is a formidable challenge requiring hundreds of lasers to be synchronized. Future lasers with intensities of the order of 10^{24} W cm^{-2} or greater created by optical parametric chirped pulse amplification may be powerful enough to propagate through meter long plasmas and produce ultra-high energy beams in the region 100 GeV–1 TeV suitable for high-energy physics experiments. Another approach is to use the existing high-energy particle beams that can also creates wakefield in plasma (Chen *et al.* 1985; Rosenzweig *et al.* 1993). A series of experiments at Stanford linear accelerator centre (SLAC) has demonstrated successfully the beam driven PWFA (Lee *et al.* 2001; Blue *et al.* 2003; Hogan *et al.* 2005). The SLAC beam is used in the experiments have intensities of the order of 10^{20} W cm^{-2}. In a particle beam driven PWFA the Coulomb force of the charged particles space charge expels the plasma electrons if it is an electron beam and pulls them in if it is a positron beam. The displaced electrons snap back to restore charge neutrality and overshoot their original positions setting up a plasma wave that trails behind the beam. If the beam is about half a plasma wavelength long the plasma wave electric field amplitude in the linear theory where $eE_p/M_e\omega_p c \ll 1$ is given by (Joshi *et al.* 2002) $eE_p = 240$ (MeV m^{-1}) $(N/(4\times10^{10}))$ $((0.6\ \text{mm})/\sigma_z)^2$, where σ_z is the bunch length and N is the number of particles per bunch. The drivers of the plasma positron wakefield experiment in the SLAC experiment was the 28.5 GeV, 2.4 ps long positron beam from the SLAC linac containing 1.2×10^{10} particles. The beam was focused at the entrance to a 1.4 m long chamber filled with a lithium plasma of density 1.8×10^{14} cm^{-3}. The first two thirds of the positron beam set up by the relativistic plasma wakefields the last third of the beam was accelerated to higher energy. The main body of the beam decelerated at a rate corresponding to 49 MeV m^{-1}, while the back of the beam containing about 5×10^8 positrons in a 1 ps slice was accelerated by 79 ± 15 MeV in the 1.4 m long plasma corresponding to an accelerating electric field of 56 MeV m^{-1}. The results are in excellent agreeing with a three-dimensional (3D) PIC simulation (Blue *et al.* 2003) which predicted a peak energy gain of 78 MeV. Very recently, Hogan *et al.* (2005) have demonstrated multi-GeV energy gain in a 10 cm long plasma using electron beams. In the beam driven wakefield experiment the beam energy is transferred from a large number of particles in the core of the bunch to a fewer number of particles in the back of the same bunch. The wakefield thus acts like a transformer with a ratio of accelerating field to decelerating field of about 1.3 in the experiment. Plasma accelerators can be used to increase the energy of the high-energy accelerator and leads the way for a possible energy doubler proposed by Lee *et al.* (2001). In the energy doubler or plasma 'afterburner' scheme (Joshi & Katsouleas 2003) a plasma wakefield accelerator using ten times shorter bunches can double the energy of a linear collider in short plasma sections of length 10 m. By reducing the positron bunch by a factor 10 the accelerating gradient can be enhanced a hundred fold to 5 GV m^{-1}, which would convert the 50 GeV SLAC linear collider into a 100 GeV machine. To sustain the luminosity at the interaction point at the nominal level of the original beam without the plasma, the reduction in the number of higher energy particles is compensated by using plasma lenses to reduce the spot size. Higher density plasma lenses are added to the afterburner design just before the interaction point. This type of plasma lens exceeds conventional magnetic lenses by several orders of magnitude in focusing gradient and was successfully demonstrated (Ng *et al.* 2001). Wakes generated by an electron or positron beam can also accelerate positrons and muons. These SLAC experiments have brought the concept of plasma accelerators to the forefront of high-energy accelerator research.

7. Photon acceleration

Photon acceleration (Wilks *et al.* 1992; Monot *et al.* 1995; Mendonça 2001; Murphy *et al.* 2005) is intimately related to short pulse amplification of plasma waves and is described by a set of equations that are similar to those in wakefield accelerators. In the last section on the wakefield amplification we noted that it was much easier to increase the amplitude of the wakefield if we used a series of photon pulses. These pulses have to be spaced in a precise manner to give the plasma wave the optimum 'kick'. If the second or subsequent photon pulses are put in a different position, 1.5 plasma wavelengths behind the first pulse, the second pulse will produce a wake that is 180° out of phase with the wake produced by the first pulse. The superposition of the two wakes behind the second pulse results in a lowering of the amplitude of the plasma wave. In fact, almost complete cancellation can take place.

The second laser pulse has absorbed some or all of the energy stored in the wake created by the first pulse. From conservation of photon number density the increase in energy of the second pulse implies that its frequency has increased. The energy in the pulse is $W = N\hbar\omega$, where N is the total number of photons in the wave packet.

A simple and rigorous account of the frequency shift can be made by using the photon equations of motion

$$\frac{\mathrm{d}\boldsymbol{r}}{\mathrm{d}t} = \frac{\partial\omega}{\partial\boldsymbol{k}}, \quad \frac{\mathrm{d}\boldsymbol{k}}{\mathrm{d}t} = -\frac{\partial\omega}{\partial\boldsymbol{r}}, \tag{7.1}$$

where $\boldsymbol{r}$ and $\boldsymbol{k}$ are the photon position and the wavevector, respectively, and $\omega = (k^2c^2 + \omega_\mathrm{p}^2(\boldsymbol{r},t))^{1/2}$ is the photon frequency.

A net frequency shift (or photon acceleration) occurs in non-stationary plasmas as well as in other time varying media. Let us take, as an example, the case of a laser (or photon) beam interacting with a relativistic ionization front with the velocity v_f. From Eqn. (36) one can readily obtain the net frequency shift of the laser beam

$$\Delta\omega = \frac{\omega_\mathrm{pe}^2}{2\omega}\frac{\beta}{1\pm\beta}, \tag{7.2}$$

where $\beta = v_\mathrm{f}/c$, and the plus and minus signs pertain for co and counter-propagation of the photons with respect to the ionization front.

8. Relativistic self-focusing and optical guiding

In the absence of optical guiding the interaction length L is limited by diffraction to $L = \pi R$, where $R = \pi\sigma^2/\lambda_0$ is the Rayleigh length and σ is the focal spot radius. This limits the overall electron energy gain in a single stage to $E_\mathrm{max}\pi R$. To increase the maximum electron energy gain per stage, it is necessary to increase the interaction length. Two approaches to keeping high-energy laser beams collimated for over a longer regions of plasma are being developed. Relativistic self-focussing can overcome diffraction and the laser pulse can be optically guided by tailoring the plasma density profile forming a plasma channel (Monot *et al.* 1995; Chen *et al.* 1998; Sarkisov *et al.* 1999).

Relativistic self-focusing uses the nonlinear interaction of the laser pulse and plasma resulting in an intensity dependent refractive index to overcome diffraction. In regions where the laser intensity is highest the relativistic mass increase is greatest; this results in a reduction of the fundamental frequency of the laser pulse. The reduction is proportional to the laser intensity. Correspondingly, the phase speed of the laser pulse will decrease in regions of higher laser intensity. This has the effect of focusing a laser beam with a radial Gaussian profile. This results in the plane wavefront bending and focussing to a smaller spot size. Relativistic self-focusing has a critical laser power threshold, P_{cr}, given by

$$P \geq P_{\mathrm{cr}} \simeq \frac{2m_{\mathrm{e}}^2 c^5 \omega_0^2}{e^2 \omega_{\mathrm{p}}^2} = 17 \frac{\omega_0^2}{\omega_{\mathrm{p}}^2} \text{ GW}. \tag{8.1}$$

The laser must also have a pulse duration that is shorter than both the collisional and ion plasma periods to avoid the competing effects of the thermal and ponderomotive self-focusing. The shape of the self-focused intense short pulses is also interesting. There is a finite time for the electrons to respond and the front of the pulse propagates unchanged. The trailing edge of the pulse compresses radially due to the nonlinear relativistic self-focusing. It is only the trailing edge of the pulse that is channelled. Several authors (Abramyan *et al.* 1992; Komashko *et al.* 1995; Cattani *et al.* 2001) have presented analytical studies of self-focusing and relativistic self-guiding of an ultrashort laser pulse in the plasma, taking into account ponderomotive force and relativistic mass variation nonlinearities. The ponderomotive force of superstrong electromagnetic fields expels electrons, thus producing 'vacuum channels' that guide the radiation and stable channeling with power higher than the critical one can take place. Specifically, Cattani *et al.* (2001) presented a simple model to investigate multifilament structures of a circularly polarized laser beam in two-dimensional planer geometry including electron cavitation that are created by the relativistic ponderomotive force. The governing equations describing filamentary structures are

$$\nabla^2 a_{\perp} + \left(1 - \frac{\alpha n}{\gamma_{\perp}}\right) a_{\perp} = 0 \tag{8.2}$$

and

$$\nabla^2 \varphi = \alpha(n-1), \tag{8.3}$$

where $\varphi = \gamma_{\perp} - 1$ if and only if $n \neq 0$, $\gamma_{\perp} = \sqrt{1 + a_{\perp}^2}$. and $\alpha = N_0/(1 - k_z^2/k^2)$. Here, we have denoted $a_{\perp} = e|\boldsymbol{A}_{\perp}|/m_{\mathrm{e}}c^2$, $N_0 = n_0/n_{\mathrm{c}}$, $n_{\mathrm{c}} = m_{\mathrm{e}}\omega^2/4\pi e^2$, $\varphi = e\phi/m_{\mathrm{e}}c^2$ and $\boldsymbol{r} = k\sqrt{1 - k_z^2/k^2}\boldsymbol{r}_{\perp}$, with $k_{\perp} \ll k_{\mathrm{z}}$. Here, $k_{\perp}$ is the transverse component of the laser wave number and k_{z} is the axial propagation constant. Eqns. (38) and (39) admit an exact solitonlike solution as well multifilament structures whose specific forms are presented by Cattani *et al.* (2001).

Another way of forming a guided laser is to create a preformed plasma with one long pulse laser, that heats and expands on a nanosecond time-scale forming a plasma density channel. Alternatively, long plasma channels could also be formed by high-voltage capillary discharges which produce the same effect.

It has been shown (Hoda *et al.* 1999) that the relativistic self-focusing in three-dimensions of a linearly polarized laser pulse propagating in an underdense plasma with power above the threshold power P_{cr} is anisotropic and evolves differently in the plane in which the pulse electric field oscillates and in the plane in which the magnetic field oscillates. The effect of the pulse polarization and of the accelerated fast electrons on the propagation anomalies 'hosing' (Duda *et al.* 1999) and 'snaking' of a high-intensity laser pulse has been investigated (Naumova *et al.* 2001) by means of fully electromagnetic relativistic two-dimensional particle-in-cell simulations. The results show that the hosing (Antonsen & Mora 1992; Sprangle *et al.* 1992) occurs in the front of the pulse, and it corresponds to low-frequency oscillations of the electron current at the sharp front of the pulse and to a periodic change of the refractive index. The hosing of the pulse may be responsible for self-trapped electrons as well as for the excitation of 'wake surface waves' (Decker *et al.* 1994) at the walls of the self-focusing channel. Snaking occurs in the later evolution of long laser pulses almost independently of their polarization.

In the present laser experiments, it is in principle feasible to reach 1–5 GeV range of energies without resorting to hybrid mechanisms. In the future, in order to achieve TeV energies substantial research must be carried out with all combinations as well as develop larger plasma channels and introduce staging. The present experiments will make it possible to judge whether a TeV machine is possible with desirable luminosities and emittances. Future high-energy accelerators may combine conventional accelerators together with a plasma afterburner to increase the beam energy. The SLAC experiments suggest that the beam can be doubled in energy if the pulse length is halved.

9. Conclusions and outlook

The present and future laser experiments, however, are very far from the parameter range of interest to high-energy physicists who require something like 10^{11} particles per pulse accelerated to TeV energies (for electrons) with a luminosity of $10^{-34}\ \mathrm{cm}^{-2}\ \mathrm{s}^{-1}$ for acceptable event rates to be achieved. The TeV energy range is >1000 times greater than a single laser accelerating stage could provide at present, even if the interaction length can be extended by laser channelling there is still going to be the requirement of multiple staging, and more energetic lasers. For a TeV beam of 10^{11} particles per pulse and a transfer efficiency of 50% would require a total of 32 kJ of laser energy per pulse, for a 100 stage accelerator. This would require 100 lasers of about 300 J each with high repetition rates. Compared to the 1–10 J lasers used in present day experiments, beam driven plasma wakefields, which contain conventional accelerators with plasma accelerators may be the way forward in the near term while more intense lasers are developed using the optical parametric CPA process.

For laser plasma accelerators, the next milestone to be achieved is the GeV energy levels with good beam quality. Electrons in this energy range are ideal as a driver for free electron lasers (FEL). At higher energies, predicted to be several GeV, it is possible to produce an X-ray FEL capable of biological investigations around the water window. Furthermore, a high-intensity laser pulse interacting with plasma can produce intense proton beams. Future plasma accelerators may produce 100 GeV, the closest experiment so far to this range is the SLAC afterburner. The TeV range although far in the distance remains a desirable goal.

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