Plasma Physics and Controlled Fusion. Vol. 29, No. 1, pp. 115 to 118, 1987
Printed in Great Britain

0741-3335/87$3.00+ .00
IOP Publishing Ltd. and Pergamon Journals Ltd.

RESEARCH NOTE

Saturation of Plasma Beat Waves by Collisional Damping

R. BINGHAM, R. A. CAIRNS,* A. E. DANGOR,† R. G. EVANS, C. J. MCKINSTRIE‡
and W. B. MORI§
Rutherford Appleton Laboratory, Chilton, Didcot, OX11 0QX, Oxford
* Mathematical Institute, University of St Andrews, St Andrews KY16 9SS
† Physics Dept. Imperial College, Prince Consort Road, London
‡ Los Alamos National Laboratory, Los Alamos, New Mexico
§ Physics Dept. UCLA Los Angeles, California

(*Received* 24 *April* 1986; *and in revised form* 26 *August* 1986)

Abstract—We analyse the saturation mechanism for the electrostatic plasma wave excited by two electromagnetic waves in a hydrogen plasma, and show that collisional damping can significantly change the saturation level determined by the relativistic frequency shift. The results will be compared with measurements obtained from recent laboratory experiments.

RECENTLY, there has been considerable interest in the nonlinear excitation of large amplitude plasma waves by beating two laser beams in an underdense plasma (TAJIMA and DAWSON, 1979; JOSHI *et al.*, 1981) as a particle accelerator for high energy physics, proposed by TAJIMA and DAWSON (1979) and known as the 'Plasma beat wave accelerator'. The scheme depends on the generation of a large amplitude plasma wave with a phase velocity close to the velocity of light. Such a wave can be produced by beating two colinear laser beams, with frequencies and wavenumbers (ω_1, k_1) and (ω_2, k_2) in a plasma where the frequency and wavenumber of the plasma wave satisfy the following resonance conditions.

$$\begin{aligned} \omega_p &= \omega_1 - \omega_2 \\ k_p &= k_1 - k_2 \end{aligned} \tag{1}$$

where $\omega_p = (\omega_{pe}^2 + 3v_{Te}^2 k_p^2)^{\frac{1}{2}}$ is the plasma wave frequency including the thermal corrections. The laser beams exert a periodic force (the ponderomotive force) on the electrons and resonantly drive the plasma wave which consists of regions of space charge. If $\omega_p \ll \omega_{1,2}$ then the phase velocity of the plasma wave $v_{ph} = \omega_p/k_p$ is equal to the group velocity of the laser beams $v_g = c(1 - \omega_{pe}^2/\omega_{1,2}^2)^{\frac{1}{2}}$ which is almost equal to the speed of light c in an underdense plasma. Particles which are injected into the beat wave region with a velocity comparable to v_{ph} can gain more energy from the longitudinal electric field of the plasma wave.

Previous studies (ROSENBLUTH and LIU, 1972; TANG *et al.*, 1984) on the nonlinear behaviour of the large amplitude plasma wave have concentrated on the relativistic effects being the only saturation mechanism for the plasma wave. The relativistic mass increase of the plasma electrons has the effect of reducing the natural frequency of oscillation of the wave which ultimately destroys the phase locking and hence provides a saturation mechanism.

In this paper we will not only include the relativistic correction factor but also the effect of plasma wave damping. The saturation level then reached by the plasma beat wave is shown to agree with recent experiments which demonstrates that collisional damping of the plasma beat wave is the dominant saturation mechanism for certain plasma and laser parameters. In particular for the experimental conditions of plasma densities $n_0 \simeq 10^{17}$ cm^{-3} and temperatures in the range 5–50 eV, collisional damping of the plasma wave determines the saturation level for laser intensities whose quiver velocity $v_{osc} \lesssim 10^{-2}$ c. We also require that $v_{osc} \ll v_{Te}$ and $v_e \ll v_{Te}$ where v_{Te} is the electron thermal speed and v_e is the electron quiver velocity due to the plasma wave these assumptions allow us to use the normal definition for the collision frequency. If these conditions are violated then the collision frequency becomes modified and we have to use an effective collision frequency which is a function not only of the electron thermal velocity but also of the electron velocity in the wave fields.

We consider an unmagnetised plasma with two parallel propagating electromagnetic waves with electric fields given by

$$\mathbf{E} = \tfrac{1}{2}(\mathbf{E}_j(\mathbf{x},t)\exp\{i(\mathbf{k}_j.\mathbf{x} - \omega_j t)\} + \mathrm{c.c}) \tag{2}$$

with polarization electric field along the y-axis with frequencies ω_j much greater than the plasma frequency $\omega_{pe} = \left(\frac{n_0 e^2}{m_e \varepsilon_0}\right)^{\frac{1}{2}}$. The normalized quiver velocities are $\alpha_j = eE_j/m_e\omega_j c, j = 1,2$.

The equations describing the behaviour of the electrons are the relativistic fluid equations and Poisson's equation,

$$\frac{\partial n_e}{\partial t} + \nabla.(n_e \mathbf{v}_e) = 0 \tag{3}$$

$$(\frac{\partial}{\partial t} + \mathbf{v}_e.\nabla + \nu_{ei})\gamma \mathbf{v}_e + \frac{3KT_e}{n_0 m_e}\nabla n_e = -\frac{e}{m_e}(\mathbf{E} + \mathbf{v} \times \mathbf{B}) \tag{4}$$

$$\nabla.\mathbf{E} = \frac{e}{\varepsilon_0}(n_e - n_0) \tag{5}$$

where $\gamma = (1 - (v_e/c)^2)^{-\frac{1}{2}}, n_0$ is the plasma density and ν_{ei} is the electron ion collision frequency. The ions cannot respond to forces acting at the electron plasma frequency and are therefore neglected. However, if we wish to study the stability of the large amplitude plasma wave with respect to wave decay and modulational instabilities due to the ponderomotive force of the plasma wave the ion dynamics must be included.

Using equations (1–5), assuming $\mathbf{v}_e^2/c^2 \ll 1$ in the Lorentz factor and introducing slowly varying amplitudes to describe the nonlinear behaviour of the laser fields and the plasma density perturbation,

$$\mathbf{E}_{1,2} = \mathrm{Re}\, \mathbf{E}_{1,2}(\mathbf{x},t)\exp\{i(k_{1,2}.\mathbf{x} - \omega_{1,2}t)\} \tag{6}$$

$$n_e = \mathrm{Re}\, N(\mathbf{x},t)\exp\{i\mathbf{k}_p.\mathbf{x}\} \tag{7}$$

where $k_p = k_1 - k_2$. Notice that we have not separated the linear time scale from the total time variation for the plasma density frequency perturbation since this mode can be strongly perturbed. The equation for the density perturbation is then

$$\left(\frac{\partial^2}{\partial t^2} + \nu_{ei}\frac{\partial}{\partial t} + \omega_p^2\right)N(t) = \frac{3}{8}\omega_{pe}^2\frac{|N|^2}{n_0^2}N - \frac{n_o}{2}\omega_{pe}^2\alpha_1,\alpha_2 \mathrm{e}^{-i\delta t} \tag{8}$$

where $\omega_p = (\omega_{pe}^2 + 3v_{Te}^2k_p^2)^{\frac{1}{2}} \approx \omega_{pe}$ and $\delta = \omega_1 - \omega_2$. Equation (8) is the Duffing equation (DAVIS, 1962) whose solutions are well known. If we drop the damping term $\nu_{ei}\frac{\partial v_e}{\partial t}$ then we get the equation ROSENBLUTH and LIU (1972) solved for no pump depletion i.e. $\alpha_1 = \alpha_2 =$ const. The solution for $N(t)$ in this case being given by

$$A(t) = A(\mathrm{o}) + \frac{1}{4}\alpha_1,\alpha_2\omega_p t \tag{9}$$

where $A(t) = N(t)/n_0$. The amplitude saturates well before reaching the wave breaking limit of $A(t) = 1$ as a result of the nonlinear frequency shift introduced by the relativistic correction. ROSENBLUTH and LIU (1972) show that the wave saturates at a value of A_{MAX} given by

$$A_{\mathrm{MAX}} = \left(\frac{16}{3}\alpha_1 \cdot \alpha_2\right)^{\frac{1}{3}} \tag{10}$$

A_{MAX} versus $\alpha_1\alpha_2$ is shown in Fig. 1.

When the time to reach saturation is greater than a collision period the effect of collisions should be included. In the case where collisions are included then equation (8) can be solved by the following method (LANDAU and LIFSHITZ, 1969). Neglect the nonlinear term $\frac{3}{8}\omega_p^2\frac{|N|^2 N}{n_0^2}$ initially and look for solutions in the form $A(t) = B\mathrm{e}^{-i\delta t}$ substituting into equation (8) we obtain the following equation for B.

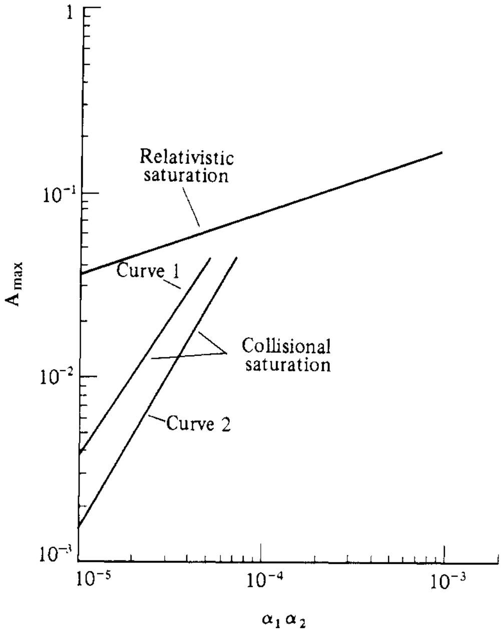

FIG. 1.—Saturation levels as a function of laser quiver velocity for the electron plasma wave determined by relativistic detuning and collisional wave damping. Curve 1 represents the collisional saturation including the modification due to large v_{osc} for a plasma density $n_0 = 10^{17}$ cm^{-3} and temperature $T_e \simeq 32$ eV, curve 2 represents a collisional saturation including the modification due to large v_{osc} for a plasma with density $n_0 = 10^{17}$ cm^{-3} and temperature $T_e \simeq 16$ eV.

$$B = \frac{1}{8}\frac{\omega_p\alpha_1\alpha_2}{(\varepsilon - i\nu_{ei}/2)} \tag{11}$$

where $\varepsilon = \omega_p - \delta$, assumed small i.e. $\delta \approx \omega_p$.
Writing $B = be^{i\varphi}$ we have

$$b^2(\varepsilon^2 + \nu_{ei}^2/4) = \frac{1}{64}\omega_p^2\alpha_1^2\alpha_2^2 \tag{12}$$

and

$$\tan\varphi = \nu_{ei}/2\varepsilon \tag{13}$$

from (13) we see that the plasma oscillation always lags behind the driving force. We now include the cubic nonlinearity which results in the appearance of an amplitude dependence of the eigenfrequency, which we now write as $\omega_p + \kappa b^2$ where $\kappa = -\frac{3}{8}\omega_p^2$, κ is negative for the relativistic correction, (which would give zero frequency as $v_e \longrightarrow c$) we now replace ω_p by $\omega_p + \kappa b^2$ such that $\delta - \omega_p$ now becomes $\delta - \omega_p - \kappa b^2$ and the equation for b is

$$b^2[(\varepsilon - \kappa b^2)^2 + \nu_{ei}^2/4] = \frac{1}{64}\omega_p^2\alpha_1\alpha_2^2. \tag{14}$$

LANDAU and LIFSHITZ (1969) give a full discussion on the shape of the curve b versus ε. The maximum value of b obtained from equation (14) occurs when $\varepsilon = \kappa b^2$ and is given by

$$b_{\text{MAX}} = A_{\text{MAX}} = \frac{1}{4}\frac{\omega_p}{\nu_{ei}}\alpha_1\alpha_2. \tag{15}$$

For the situation where $v_{\text{osc}} \gtrsim v_{Te}$ the collision frequency becomes modified and ν_{ei} has to be multiplied by a factor $(1 + v_{\text{osc}}^2/v_{Te}^2)^{-\frac{3}{2}}$ with the result that equation (15) becomes

$$A_{\text{MAX}} = \frac{1}{4}\frac{\omega_p}{\nu_{ei}}\alpha_1\alpha_2(1 + v_{\text{osc}}^2/v_{Te}^2)^{\frac{3}{2}}. \tag{16}$$

The slope of A_{MAX} versus $\alpha_1\alpha_2$ (due to collisional damping) is also shown in Fig. 1. We have plotted two curves for the maximum amplitude of the plasma wave corresponding to the two experiments (Clayton *et al.*, 1985; Dangor, private commun.) mentioned earlier in the paper. Curve 1 represents the expected saturation value from the UCLA experiment (Clayton *et al.*, 1985) where $n_0 = 10^{17}$ cm^{-3}, $T_e = 32$ eV and $\alpha_1\alpha_2 \approx 7 \times 10^{-4}$. The experimentally measured value for A_{MAX} is about 0.08 which is close to both the relativistic saturation leval determined by equation (10) and also the collisional saturation level given by equation (16). In the RAL experiment, (Dangor, private commun.), where densities and temperature ($n_0 = 10^{17}$ cm^{-3}, $T_e = 16$ eV) were similar to the UCLA experiment but the laser intensity was such that the expected plasma wave amplitude determined by relativistic saturation is 0.01–0.03 which is well within the sensitivity of the detection system which can detect plasma waves as low as 5×10^{-3} in amplitude.

However, the RAL experiment did not detect any plasma waves which suggests that the amplitude of these waves was lower than 5×10^{-3}. Curve 2 represents the expected amplitude level in the RAL experiment (determined by damping). The expected amplitude level determined from equation (16) is about 10^{-3} which is below the sensitivity of the experiment.

In conclusion we have shown that if the saturation mechanism was purely due to the relativistic frequency shift of the plasma wave then the RAL experiment should have detected it. Even if we take into account finite rise time effects in the laser beam we would still be well within the sensitivity level of the experimental method. Also with the 300 ps pulse length there is sufficient time for the waves to grow to their final level. From equation (9) we estimate the growth time for relativistic saturation to be about 60 ps. All the evidence points to other factors which saturate the wave and we expect collisional damping to be dominant for the experiments carried out in the high density, low temperature regime and at modest laser intensities such that $\alpha_1\alpha_2 < 10^{-4}$. The full effects of the nonlinear solutions to equation (8) together with pump depletion will be reported in a later paper.

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