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TOPICAL REVIEW

Plasma based charged-particle accelerators

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Abstract
Studies of charged-particle acceleration processes remain one of the most important areas of research in laboratory, space and astrophysical plasmas. In this paper, we present the underlying physics and the present status of high gradient and high energy plasma accelerators. We will focus on the acceleration of charged particles to relativistic energies by plasma waves that are created by intense laser and particle beams. The generation of relativistic plasma waves by intense lasers or electron beams in plasmas is important in the quest for producing ultra-high acceleration gradients for accelerators. With the development of compact short pulse high brightness lasers and electron positron beams, new areas of studies for laser/particle beam-matter interactions is opening up. A number of methods are being pursued vigorously to achieve ultra-high acceleration gradients. These include the plasma beat wave accelerator mechanism, which uses conventional long pulse (∼100 ps) modest intensity lasers ($I \sim 10^{14}$–10^{16} W cm^{-2}), the laser wakefield accelerator (LWFA), which uses the new breed of compact high brightness lasers (<1 ps) and intensities $>10^{18}$ W cm^{-2}, the self-modulated LWFA concept, which combines elements of stimulated Raman forward scattering, and electron acceleration by nonlinear plasma waves excited by relativistic electron and positron bunches. In the ultra-high intensity regime, laser/particle beam–plasma interactions are highly nonlinear and relativistic, leading to new phenomena such as the plasma wakefield excitation for particle acceleration, relativistic self-focusing and guiding of laser beams, high-harmonic generation, acceleration of electrons, positrons, protons and photons. Fields greater than 1 GV cm^{-1} have been generated with particles being accelerated to 200 MeV over a distance of millimetre. Plasma wakefields driven by positron beams at the Stanford Linear Accelerator Center facility have accelerated the tail of the positron beam. In the near future, laser plasma accelerators will be producing GeV particles.

1. Introduction

Plasma is an attractive medium for particle acceleration [1] because of the high electric field it can sustain with studies of acceleration processes remaining one of the most important areas of research in both laboratory space and astrophysical plasmas. The subject is also vast, making it almost impossible to cover all aspects. Our objective here is to concentrate mainly on the fundamental physics of particle acceleration by relativistic plasma waves that are generated, for example, by intense laser or particle beams. In a plasma-based accelerator [2], particles gain energy from a longitudinal plasma wave. To accelerate particles to relativistic energies, the plasma waves have to be sufficiently intense, with a phase speed close to the speed of light c in vacuum.

A number of laboratory schemes are used to generate intense plasma waves that can accelerate charged particles. The most successful are those based on large amplitude relativistic plasma waves generated by lasers, first proposed by Tajima and Dawson [2]. Particle acceleration by relativistic electron plasma waves has been demonstrated in a number of experiments, the most recent [3–5] producing more than hundred MeV electrons in distances of about 1 mm. Resulting accelerating fields as high as 1 GV cm^{-1} have been achieved in these experiments. Note that the maximum accelerating field in high energy accelerators is of the order of 20 MV m^{-1}.

The rapid advance in laser technology on femtosecond pulse amplification in the mid-1980s initiated the development of compact terawatt and petawatt laser systems with ultra-high intensities ($\geqslant 10^{18}$ W cm^{-2}), with modest energies ($\leqslant 100$ J) and short sub-picosecond pulses ($\leqslant 1$ ps). This new breed of lasers whether they be excimer, dye, gas or glass are commonly referred to as T^3 (tabletop-terawatt) lasers (T cubed). These lasers have now made it possible to study laser–plasma interactions at ultra-high intensities or high brightness where the laser–electron interaction becomes highly nonlinear and relativistic resulting in a wide variety of interesting phenomena such as (i) plasma wakefield excitation; (ii) relativistic self-focusing and guiding of lasers in plasma channels; (iii) relativistic self-phase modulation; (iv) photon acceleration; (v) proton acceleration; (vi) harmonic generation, (vii) ultra-high magnetic field generation, etc. In the past, several authors [6–13] described the essential physics of high energy plasma accelerators, as well as the progress that were made at that time.

Short pulse high brightness lasers are not the only way to produce relativistic waves in plasmas. For a number of years some groups have been using the beat wave process to generate intense relativistic electron plasma waves in preformed plasmas. The beat wave process relies on using two long laser pulses ($\leqslant 100$ ps) of moderate intensity $I \simeq 10^{15}$ W cm^{-2} co-linearly injected into a low density plasma such that the plasma frequency equals the difference frequency of the two laser beams. Under such conditions, large amplitude relativistic plasma waves are generated with a phase velocity equal to the group velocity of the laser beam, which is close to the speed of light.

The generation of relativistic plasma waves using the laser wakefield or the beat wave mechanism was first proposed by Tajima and Dawson in 1979 [2] for producing an ultra-high gradient plasma accelerator. But it was not until 1994 that Dangor *et al* [15], Kitagawa *et al* [16], Clayton *et al* [17] and Everett *et al* [18] demonstrated that high-energy injected electrons could be trapped and accelerated to significant energies by relativistic plasma waves. Recently, Walton *et al* [19] have also observed the generation of relativistic plasma waves by a high-intensity short-pulse beat wave. The main emphasis in the beat wave experiments was to demonstrate particle acceleration by the relativistic plasma wave. In these experiments, the maximum accelerating field gradient E was limited by the wave breaking, which occurs for a cold plasma when the plasma wave density perturbation δn equals the mean plasma number density n_0. Wavebreaking occurs when higher harmonics created due to nonlinear processes distort an initially sinusoidal wave into a topsided triangular waveform with a steep leading edge, the gradient of which eventually becomes infinite. The mathematical solution to this problem becomes multi-valued, and the wave turns over and 'breaks' like an ocean wave on the beach, converting the wave energy into thermal energy of the particle. In reality, the maximum wave amplitude is less than this and is determined mainly by relativistic de-tuning due to the relativistic electron mass increase. The generation of plasma fields due to the interaction between two oppositely propagating short laser pulses in an underdense plasma has been theoretically considered by Gorbunov and Frolov [20].

In both the proposed mechanisms for producing ultra-high accelerating electric fields, the electron quiver velocity in the plasma wave, defined as $v_{\rm osc} = eE/m_e\omega_p$, is greater than the speed of light, where e is the magnitude of the electron charge, E is the electric field of the plasma wave, m_e is the electron rest mass, and ω_p is the electron plasma frequency. The electron quiver velocity in the field of the laser (defined as $v_{\rm osc} = eE/m_e\omega_L$, where E is the laser electric field and ω_L is the laser frequency) for the beat wave case is not relativistic, whereas in the high brightness laser field it is relativistic. This makes a considerable difference to the physics of nonlinear interactions between the two processes. Alternatively, instead of laser beams, charged particle beams are also used for generating large amplitude plasma waves. Here, multi-GeV electron and positron beams are used to further enhance the energy of accelerated particles [21–24] because of the incredibly strong electric fields that are induced in the plasma by space charge effects.

Conventional accelerators use radio frequency (RF) waves to accelerate particles; these devices are limited by the breakdown voltage of the waveguide structure to maximum accelerating fields of about 50 MV m^{-1}. The accelerating gradients and focusing strength that have been demonstrated in plasma accelerators are orders of magnitude higher of the order of GV m^{-1}. The success of plasma accelerators is plotted in figure 1, known as the

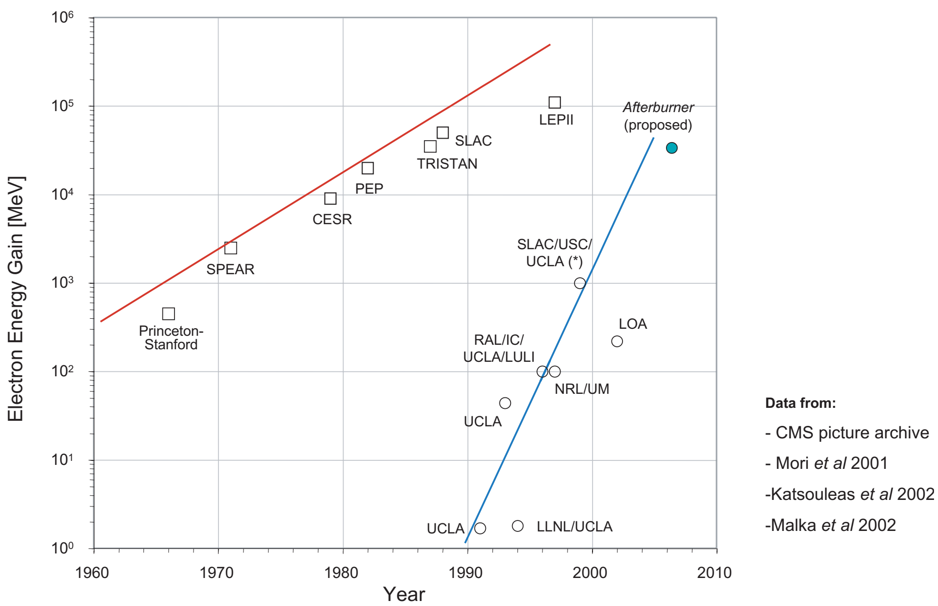

Figure 1. The Livingston curve showing the progress of particle physics accelerators over the decades.

'Livingston curve' that charts the progress of particle physics accelerators over the decades. Since the early 1990s significant progress has been made in both laser and particle beam driven plasma accelerators. The accelerating gradient although important is not the only parameter necessary to build a successful accelerator, Luminosity and emittance are two others that have to match or be better than that in conventional accelerators. For example, the next linear colliders being planned will have a luminosity in the region 10^{34} erg cm^{-2} s^{-1}, which is beyond the present capability of plasma accelerators. Plasma accelerators are ideal for providing compact, short pulses and they may also be useful in increasing the energy of conventional accelerators using the afterburner concept.

In this paper, we review the underlying physics and the present status of plasma based charged particle accelerators that are of significant importance [23–26] in high energy physics and medicine. Specifically, we shall concentrate on the generation of large amplitude electron plasma waves by the beat wave and laser wakefield mechanisms, and by particle beams. We will also touch on topics such as photon acceleration, relativistic self-focusing and plasma guiding. The laser guiding process is of particular importance since intense non-diverging laser beams will be useful in delivering an extended interaction region for plasma wave accelerators, which are limited at present to a Rayleigh length.

This paper is organized in the following fashion. In section 2, we present the physics of relativistic plasma wave acceleration. In section 3, we discuss the plasma beat wave accelerator (PBWA). The laser wakefield accelerator (LWFA) scheme is presented in section 4. The model equations for laser wake field excitation are described in section 5. Section 6 is devoted to the mechanism of self-modulated wakefields due to forward Raman scattering. Section 7 deals with the physics of photon acceleration in plasmas. Section 8 contains materials pertaining to relativistic self-focusing and optical guiding of intense laser beams. Finally, conclusions and outlook are given in section 9.

2. Relativistic plasma wave acceleration

Particle acceleration by relativistic plasma waves has gained a lot of interest lately due to the rapid advances in laser technology and the development of compact terawatt and petawatt laser systems with ultra-high intensities ($\geqslant 10^{18}$ W cm^{-2}), modest energies ($\leqslant$100 J) and short, sub-picosecond pulses ($\leqslant$1 ps). The strength of the electric field at the focus of these high-power, short-pulse lasers $E_\perp$ is directly related to the laser intensity as $eE_\perp \approx 30\sqrt{I}$ GeV cm^{-1}. The electric field $E_\perp$ of a laser, whose intensity I is 10^{18} W cm^{-2}, is 30 GV cm^{-1}, at 10^{21} W cm^{-2} the field is $\sim$1 TV cm^{-1}. Direct use of the laser field for particle acceleration is not straightforward. Since the electric field of the laser is perpendicular to the propagation direction, the maximum energy gain is limited by the distance the particle moves across the wavefront before the electric field changes sign. However, the situation changes when one uses a plasma into which laser energy can be coupled. Plasma as a medium for particle acceleration has a number of advantages. It has no electrical breakdown limit like conventional accelerating structures, which are limited to a maximum field strength of less than 1 MV cm^{-1}. A plasma supports longitudinal plasma waves, which oscillate at the plasma frequency $\omega_p \equiv (4\pi n_0 e^2/m_e)^{1/2}$, where n_0 and m_e are the electron number density and the mass, respectively. In these waves the plasma electrons oscillate back and forth at ω_p irrespective of the wavelength. Therefore, these waves can have arbitrary phase speed, $v_{\rm ph}$; relativistic plasma waves have $v_{\rm ph} \lesssim c$. The electric field E of relativistic plasma waves with an oscillatory density n_1, can be estimated from Poisson's equation, yielding $E = \epsilon\sqrt{n}$ V cm^{-1}, where n is the plasma number density in cm^{-3} and ϵ is the plasma wave amplitude or fractional density bunching n_1/n_0. For a plasma density of 10^{19} cm^{-3} accelerating gradients of 1 GV cm^{-1} are possible, which is more than a thousand times larger than in conventional accelerators. This aspect of plasma accelerators is what makes them a very attractive alternative to conventional accelerators. The high accelerating gradients allows the possibility of building compact 'tabletop' accelerators rather than multi-kilometre sized present-day accelerators. In their seminal paper on plasma based accelerators, Tajima and Dawson [2] showed how intense short pulse lasers with a pulse length equal to half the plasma wavelength could generate large amplitude relativistic longitudinal plasma waves. This scheme has become known as the LWFA. Alternative schemes to excite plasma waves using large laser pulses are: (i) the PBWA [2] where two long laser pulses with a frequency separation equal to ω_p beat together in a plasma to excite the plasma wave by resonance; (ii) the Raman forward scattering (RFS) instability, where one long intense laser pulse is used; this is now called the self-modulated LWFA scheme [27]. Alternatively, instead of using lasers short relativistic particle beams can also excite large amplitude relativistic longitudinal plasma waves. In a beam driven plasma wakefield accelerator (PWFA) a large amplitude relativistic plasma wave is excited by a short (in comparison to the plasma wavelength), high charge relativistic beam. The Coulomb force of the beam's space charge expels plasma electrons, which rush back in after the beam has passed setting up a plasma oscillation [28]. Both electrons or positrons can be used to excite the plasma wakefield; in the case of positrons, electrons from the background plasma are pulled in by the bunch these electrons overshoot and set up the plasma oscillation. Tajima and Dawson [2] showed that the maximum energy gain ΔW of a particle in relativistic plasma waves with $v_{\text{ph}} \leqslant c$ is

$$\Delta W = 2\epsilon\gamma^2 m_e c^2, \tag{1}$$

where γ is the Lorentz factor associated with the phase velocity of the plasma wave $\gamma = (1 - v_{\text{ph}}^2/c^2)^{1/2}$. The phase velocity of the plasma wave is equal to the group velocity v_g of the laser in the plasma, namely

$$v_{\text{ph}} = v_g = c\left(1 - \frac{\omega_p^2}{\omega^2}\right)^{1/2} \approx c\left(1 - \frac{\omega_p^2}{2\omega^2}\right),$$

where ω is the laser frequency; therefore, $\gamma = \omega/\omega_p$ and the maximum energy gain is

$$\Delta W = 2\epsilon\frac{\omega^2}{\omega_p^2} m_e c^2. \tag{2}$$

It is clear that for given values of ω there is a trade-off to be considered in choosing ω_p. From the group velocity v_g we see that a low value of ω_p is required to minimize the phase slip of extremely relativistic electrons with respect to the wave, whereas a high value of ω_p is necessary to maximize the accelerating field E. We would like to maximize E by increasing ω_p, but this minimizes the energy gain ΔW due to phase slip. As the electron accelerates it slips forward in phase and eventually outruns the useful part of the accelerating field. Knowing the plasma wavelength and the velocity difference between the plasma wave and particle $\Delta v \simeq c - v_g$, it is possible to determine the phase slip, $\delta = (\omega_p^2/\omega^2)(Lk_p/2)$, where L is the length of the acceleration stage. Clearly the maximum phase shift cannot exceed π. It can be shown that $5\pi/8$ is a preferable figure, since near zero and π the acceleration is small and inefficient. However, only half this range, i.e. $5\pi/16$ is available. The reason for this is that the plasma is bounded in the transverse direction and, consequently, there is a radial field in quadrature with the longitudinal field, which produces a strong defocusing force over the first half of the accelerating phase and a strong focusing force over the second half. This limits the acceptable range to only $5\pi/16$. To prevent phase slip, the accelerator must be split into stages of length $\lambda_p(\omega^2/\omega_p^2)(5/16)$. The maximum energy gain occurs over a distance

$L(= \Delta W/eE = 2\gamma^2 c/\omega_p)$, which is the limit of the de-phasing length. These three schemes are based on the generation of coherent large amplitude electron plasma waves travelling with a phase speed close to the speed of light.

3. Plasma beat wave accelerator

In the PBWA a relativistic plasma wave is generated by the ponderomotive force of two lasers separated in frequency by the plasma frequency, such that the energy and momentum conservation relations are satisfied, namely $\omega_1 - \omega_2 = \omega_p$ and $k_1 - k_2 = k_p$, where $\omega_{1,2}$, and $k_{1,2}$ are the frequencies and wavenumbers of the two lasers, respectively, and k_p is the plasma wave wavenumber.

The beat pattern can be viewed as a series of short light pulses, each $\pi c/\omega_p$ long, moving through the plasma at the group velocity of light, which for $\omega_{1,2} \gg \omega_p$ is close to c. The plasma electrons feel the periodic ponderomotive force of these pulses. Since this frequency difference matches the natural oscillation frequency of the electron plasma wave, ω_p, the plasma responds resonantly to the ponderomotive force and large amplitude plasma waves are built up.

If $\omega_p \ll \omega_{1,2}$ then the phase velocity of the plasma wave $v_{\mathrm{ph}} = \omega_p/k_p = (\omega_1 - \omega_2)/(k_1 - k_2) = \Delta\omega/\Delta k$ equals the group velocity of the laser beams $v_g = c(1 - \omega_p^2/\omega_{1,2}^2)^{1/2}$, which is almost equal to c in an underdense plasma. Particles that are injected into the beat wave region, with a velocity comparable to the phase velocity of the electron plasma wave, can gain more energy from the longitudinal electric field. Because ω_1 is close to ω_2 and much larger than ω_p, the Lorentz factor γ_p associated with the beat waves is

$$\gamma_p = \left(1 - \frac{v_{\mathrm{ph}}^2}{c^2}\right)^{-1/2} = \frac{\omega_{1,2}}{\omega_p} \gg 1. \tag{3}$$

The beat wave process is related to stimulated RFS (SRFS). Stimulated Raman scattering is the terminology used in plasma physics for the scattering of electromagnetic waves by longitudinal electron plasma waves. If the scattered electromagnetic wave propagates in the same direction as the incident electromagnetic wave, we refer to this as forward scattering. Electron plasma waves are also sometimes referred to as Langmuir waves after E. Langmuir who was the first to discover them. The general equations describing the beat wave and SRFS are similar. It is sufficient to analyse the problem of plasma wave growth and saturation using the relativistic fluid equations for electrons—Maxwell's and Poisson's equations—namely

$$\frac{\partial n_e}{\partial t} + \nabla \cdot (n_e \boldsymbol{v}_e) = 0, \tag{4}$$

$$\left(\frac{\partial}{\partial t} + \boldsymbol{v}_e \cdot \nabla\right)(\gamma_e \boldsymbol{v}_e) = -\frac{e}{m_e}\left(\boldsymbol{E} + \frac{1}{c}\boldsymbol{v}_e \times \boldsymbol{B}\right) - \frac{3k_B T_e}{n_0 m_e}\nabla n_e, \tag{5}$$

$$\nabla \times \boldsymbol{E} = -\frac{1}{c}\frac{\partial \boldsymbol{B}}{\partial t}, \tag{6}$$

$$\nabla \times \boldsymbol{B} = \frac{4\pi}{c}\boldsymbol{J} + \frac{1}{c}\frac{\partial \boldsymbol{E}}{\partial t} \tag{7}$$

and

$$\nabla \cdot \boldsymbol{E} = 4\pi e(n_e - n_0), \tag{8}$$

where n_e is the electron number density, $\boldsymbol{v}_e$ is the electron fluid velocity, $\gamma_e = (1 - v_e^2/c^2)^{-1/2}$ is the relativistic gamma factor, $\boldsymbol{E}$ and $\boldsymbol{B}$ are the electric and magnetic fields, respectively, k_B is the Boltzmann constant, and T_e is the electron temperature. Note that to study the stability of large amplitude plasma waves with respect to wave decay, and modulational instabilities due to the ponderomotive force of the plasma wave, the ion dynamics must also be included within the framework of a kinetic treatment [26].

For the plasma beat wave study, we use equations (4)–(8), with $v_e^2/c^2 \ll 1$ in the Lorentz factor and introduce slowly varying amplitudes to describe the nonlinear behaviour of the laser field represented by

$$\boldsymbol{E}_{1,2} = \mathrm{Re}\,\boldsymbol{E}'_{1,2}(x,t)\exp[\mathrm{i}(k_{1,2}x - \omega_{1,2}t)] \tag{9}$$

and takes the plasma density perturbation as

$$\delta n_e = n_e - n_0 = \mathrm{Re}\,\delta n'_e(x,t)\exp(\mathrm{i}k_p x). \tag{10}$$

Note that we have not separated the linear timescale from the total time variation for the plasma density perturbation, since this mode can be strongly nonlinear.

The equation for the plasma number density perturbation is found to be

$$\left(\frac{\partial^2}{\partial t^2} + \omega_p^2\right)\delta n_e = \frac{3}{8}\omega_p^2\left(\frac{\delta n_e}{n_0^2}\right)^2\delta n_e - \frac{n_0}{2}\omega_{\mathrm{pe}}^2\alpha_1\alpha_2\exp(-\mathrm{i}\delta t), \tag{11}$$

where $\alpha_j = eE_j/m_e\omega_j c$, $j = 1, 2$, is the normalized quiver velocity in the field of each laser and $\delta = \omega_1 - \omega_2$ is the frequency mismatch. (Note that we have dropped the primes on δn_e and $\boldsymbol{E}_j$.) Rosenbluth and Liu [30] solved equation (11) in the limit of zero pump depletion, i.e. $\alpha_1 = \alpha_2 =$ const., obtaining

$$\frac{\delta n_e(t)}{n_0} = \frac{\delta n_e(0)}{n_0} + \frac{1}{4}\alpha_1\alpha_2\omega_p t, \tag{12}$$

which shows that the plasma wave amplitude initially grows linearly with time. However, due to the second term on the right-hand side of equation (12), which is a cubic nonlinearity in δn_e (cf the first term on the right-hand side of equation (11)) and is due to the relativistic electron mass increase in the field of the Langmuir wave, the amplitude growth will slow down and saturation will eventually occur. Rosenbluth and Liu [30] showed that the wave saturated at an amplitude level well before reaching the wave breaking limit of $\delta n_e/n_0 = 1$. We have

$$\frac{\delta n_{e\,\max}}{n_0} = \left(\frac{16}{3}\alpha_1\alpha_2\right)^{1/3} \equiv \epsilon. \tag{13}$$

From equation (11), we see that the relativistic mass increase of the plasma electrons has the effect of reducing the natural frequency of oscillation. From the continuity equation one finds that the electron quiver velocity in the plasma wave is $v_{\mathrm{osc}}/c = \delta n_e/n_0$ for $\omega_0/k_0 \simeq c$. The natural frequency of oscillation of the plasma wave is reduced, and is given by

$$\omega'_p = \omega_p\left(1 - \frac{3}{8}\frac{v_{\mathrm{osc}}^2}{c^2}\right)^{1/2} \equiv \omega_p\left(1 - \frac{3}{8}\frac{\delta n_e}{n_0}\right)^{1/2}. \tag{14}$$

It was pointed out by Tang *et al* [31] that by deliberately allowing for the relativistic mass variation effect [32] and having a denser plasma such that the plasma frequency was initially larger than the laser frequency difference the plasma wave would come into resonance as it grew, allowing a larger maximum saturation value to be attained. An increase of about 50% in the saturated wave amplitude can be achieved using this technique.

The longitudinal field amplitude of these relativistic plasma waves can be extremely large with a theoretical maximum obtained from Poisson's equation, and is given by

$$E = \epsilon\sqrt{n_0} \quad (\mathrm{V\,cm^{-1}}), \tag{15}$$

where ϵ is the Rosenbluth and Liu [30] saturation value defined by equation (13). For the plasma densities of the order of 10^{19} cm^{-3} and saturated values of 30% obtained in present-day experiments, the field strength can be of the order of 10^9 V cm^{-1}, which is close to the Coulomb field of a proton E_a at a distance of the order of a Bohr radius a_0, $E_a = 5\times 10^9$ V cm^{-1}. This longitudinal field is capable of producing a GeV electron in a distance of 1 cm.

An important consideration in the beat wave scheme is to have sufficiently intense lasers such that the time to reach saturation is short compared to the ion plasma period. When the timescale is longer than the latter, the ion dynamics becomes important and the electron plasma wave becomes modulationally unstable by coupling to low-frequency ion density perturbations [33].

The growth of the plasma wave due to the beat wave mechanism is described by

$$\epsilon = \int_0^t \frac{\alpha_1\alpha_2\omega_p}{4}\,\mathrm{d}t, \tag{16}$$

where $\alpha_{1,2} = eE_{1,2}/m_e\omega_{1,2}c$ is the normalized oscillatory velocity of an electron in the laser fields $E_{1,2}$. As the electron plasma wave grows, its electric field amplitude given by equation (15) becomes large enough that the velocity of an electron oscillating in this field becomes relativistic and the plasma frequency ω_p suffers a small red shift $\Delta\omega_p = -(\frac{3}{16})\epsilon^2$ due to the relativistic increase in the electron mass. This red shift in the frequency causes the wave to saturate at

$$\epsilon_{\rm sat} = \left(\tfrac{16}{3}\alpha_1\alpha_2\right)^{1/3} \tag{17}$$

and the time for saturation is

$$\tau_{\rm sat} = \frac{8}{\omega_p}\left(\frac{2}{3}\right)^{1/3}\left(\frac{1}{\alpha_1\alpha_2}\right)^{2/3}. \tag{18}$$

Other factors which can limit the interaction or acceleration length is diffraction of the laser beams or the pump depletion. Diffraction limits the depth of focus to the Rayleigh length, which may be overcome by channelling of the laser [35, 36]. Pump depletion can be avoided by using more powerful lasers. By using intense, short pulse lasers, ion instabilities, such as the stimulated Brillouin and plasma modulational instabilities [13], can be avoided. A number of experiments have been carried out which demonstrate that the theoretical estimates are in very good agreement with observations.

The experiments carried out at UCLA [17, 18] focused a two frequency carbon dioxide laser and injected a 2 MeV electron beam to the same point in a hydrogen plasma at a density of about 10^{16} cm^{-3}. The results showed that approximately 1% or 10^5 electrons of the randomly phased injected electrons are accelerated from 2 to 30 MeV in the diffraction length of about 1 cm. This corresponds to a gradient of 0.03 GV cm^{-1}. The measured amplitude of the relativistic plasma waves is 30% of its cold wavebreaking limit, agreeing with the theoretical limit given by equation (17). What is particularly significant about this experiment is that it demonstrated that the electrons were 'trapped' by the wave potential. Only trapped electrons can gain the theoretical maximum amount of energy limited by de-phasing, which occurs when the polarity of the electric field of the plasma wave seen by the accelerated electron changes sign.

A trapped electron, by definition, moves synchronously with the wave at the point of reflection in the wave potential. At this point, trapped electrons have a relativistic Lorentz factor $\gamma = (1 - v_{\rm ph}^2/c^2)^{1/2}$. As the electron continues to gain energy it remains trapped (and eventually executes a closed orbit in the wave potential). Trapping also bunches electrons. In the UCLA experiment [17], the plasma wave has a Lorentz factor of 33, which is synchronous with 16 MeV electrons. Therefore, all electrons observed above 16 MeV are trapped and move forward in the frame of the wave.

The experiment done at the Ecole Polytechnique [33] also accelerated electrons but they were limited to very small energy gains from 3 to 3.7 MeV due to the saturation of the relativistic plasma wave by the modulational instability of the plasma wave coupling to the low frequency in the acoustic mode. This instability is important for long pulses of the order of the ion plasma period $\omega_{\mathrm{pi}}^{-1}$, and it limits the wave amplitude to very small values. All beat wave experiments confirm earlier simulations [34] and theoretical work demonstrated the need to use short pulses to avoid competing instabilities.

The success of the experiments indicate that it should be possible to accelerate electrons to 1 GeV in a single stage laser plasma accelerator. In such an experiment an injected 10 MeV beam of electrons of 100 A could produce about 10^8 electrons at 1 GeV energies. The necessary laser power required is $\sim$14 TW (14×10^{12} W) with a pulse duration of 2 ps, corresponding to a laser energy of 28 J and wavelengths of 1.05 and 1.06 μm in a plasma with density 10^{17} cm^{-3} and interaction length $\simeq$3 cm. From these parameters we find that the plasma wave will saturate at a value of $\delta n/n_0 \simeq 0.45$, resulting in a field gradient $E = 0.45\sqrt{n_0} \simeq 140\,\mathrm{MV\,cm^{-1}}$ for $n_0 \simeq 10^{17}\,\mathrm{cm^{-3}}$. Assuming no self-focusing or laser guiding, this accelerating field is constant over a Rayleigh length $R \simeq \pi\theta^2/\lambda \simeq 0.34$ cm, where θ is the spot size. This results in a maximum energy gain of

$$\Delta W = eE\pi R \simeq 150\,\mathrm{MeV}. \tag{19}$$

By using a discharge channel or capillary channelling [36, 35] which can measure the interaction length with considerable accuracy, for an interaction length of about 1 cm, an energy gain of 1 GeV is possible.

One of the problems to overcome is the ion instabilities due to the small ion plasma period, which is of the order of 15 ps. To avoid ion plasma instabilities, such as the ion modulational instability, the plasma density could be reduced. This has the effect of making the laser beams appear shorter, but it also reduces the maximum accelerating gradient.

4. Laser wakefield accelerator

In the LWFA scheme, a short laser pulse, whose frequency is much larger than the plasma frequency, excites a wake of plasma oscillations (at ω_p) due to the ponderomotive force much like the wake of a motor boat. Since the plasma wave is not resonantly driven, as in the beat wave, the plasma density does not have to be of a high uniformity to produce large amplitude waves. As an intense pulse propagates through an underdense plasma ($\omega_0 \gg \omega_p$, where ω_0 is the laser frequency) the ponderomotive force associated with the laser envelope $F_{\mathrm{pond}} \simeq -(m/2)\nabla v_{\mathrm{osc}}^2$ expels electrons from the region of the laser pulse and excites electron plasma waves. These waves are generated as a result of being displaced by the leading edge of the laser pulse. If the laser pulse length ($c\tau_L$) is large compared to the electron plasma wavelength, then the energy in the plasma wave is re-absorbed by the trailing part of the laser pulse. However, if the pulse length is approximately equal to or shorter than the plasma wavelength, namely $c\tau_L \simeq \lambda_p$, the ponderomotive force excites plasma waves or wakefields with a phase velocity equal to the laser group velocity, and the energy is not re-absorbed. Thus, any pulse with a sharp rise or a sharp fall on a scale of c/ω_p will excite a wake. With the development of high brightness lasers the laser wakefield concept [2] has now become a reality. The focal intensities of such lasers are $\geqslant 10^{19}\,\mathrm{W\,cm^{-2}}$, with $v_{\mathrm{osc}}/c \geqslant 1$, which is the strong nonlinear relativistic regime. Any analysis must, therefore, be in the strong nonlinear relativistic regime and a perturbation procedure is invalid.

The maximum wake electric field amplitude generated by a plane polarized laser pulse has been given by Sprangle *et al* [37] in the one-dimensional limit as

$E_{\rm max} = 0.38(v_{\rm osc}/c)^2(1 + v_{\rm osc}^2/2c^2)^{-1/2}\sqrt{n_0}\,{\rm V\,cm^{-1}}$ for $v_{\rm osc}/c \sim 4$, and $n_0 = 10^{18}\,{\rm cm^{-1}}$, $E_{\rm max} \approx 2\,{\rm GV\,cm^{-1}}$, and the time to reach this amplitude level is of the order of the laser pulse length. There is no growth phase as in the beat wave situation, which requires many plasma periods to reach its maximum amplitude.

To get larger wave amplitudes, several groups have suggested [38–41] using multiple pulses with varying time delay between pulses. Conclusive experiments [3, 6, 43–46, 48–53] have been carried out to demonstrate the excitation of the plasma wakefield. The group at the Ecole Polytechnique successfully demonstrated gains of about several MeV due to the wakefield [46, 47], which is obtained by means of sequences of short laser pulses. The most impressive results for LWFA have come from Malka's group at LOA [6], where electron energies up to 200 MeV have been observed in plasmas as small as a millimetre. The experiment was conducted in the strongly nonlinear regime using a Ti : Sa laser operating at 10 Hz. The laser delivered energies up to 1 J on target with a pulse length of 30 fs, producing a focused intensity of $3\times10^{18}\,{\rm W\,cm^{-2}}$ corresponding to a normalized vector potential, $a_0 = eA/m_0c^2$, of 1.2. The density of plasma chosen was between 2×10^{19} and $6\times10^{19}\,{\rm cm^{-3}}$, which corresponds to a period of 25–14 fs similar to the pulse length of the laser. The observed electron spectrum can be characterized by two distinct regions: below 130 MeV it has a longitudinal 'effective' temperature of 18 MeV, but at higher energies, extending from 130 to 200 MeV, a plateau forms with a significant number of electrons in this high energy tail. Estimates of the maximum energy indicate that these higher energy electrons are close to the cold wave breaking limit of 250 MeV. The experiment also measured laser pulse distortion as a result of the nonlinear interaction between the laser and the plasma in setting up the ultra-strong relativistic plasma wave.

5. Equations describing laser wakefield

To understand the laser wakefield excitation mechanism, it is sufficient to use a model based on the one fluid, cold relativistic hydrodynamics and Maxwell equations together with a 'quasi-static' approximation, a set of two coupled nonlinear equations describing the self-consistent evolution in one-dimension of the laser pulse vector potential envelope and the scalar potential of the excited wakefield. Accordingly, one starts with the relativistic electron momentum equation

$$\frac{\partial \boldsymbol{p}}{\partial t} + v_z\frac{\partial \boldsymbol{p}}{\partial z} = -e\left(\boldsymbol{E} + \frac{1}{c}\boldsymbol{v}\times\boldsymbol{B}\right), \tag{20}$$

where the relativistic momentum is

$$\boldsymbol{p} = m_e\gamma\boldsymbol{v}, \qquad \text{with } \gamma = \left(1 + \frac{p^2}{m_e^2c^2}\right)^{1/2}$$

and $\boldsymbol{v}$ being the electron velocity. In equation (20), we have assumed that all quantities only depend on z and t; z is the direction of propagation of the (external) laser pump. The electromagnetic fields are given by

$$\boldsymbol{E} = -\frac{1}{c}\frac{\partial \boldsymbol{A}_\perp}{\partial z} - \hat{z}\frac{\partial\phi}{\partial z}, \qquad \boldsymbol{B} = \nabla\times\boldsymbol{A}_\perp, \qquad \boldsymbol{A}_\perp = \hat{x}A_x + \hat{y}A_y, \tag{21}$$

where $\boldsymbol{A}_\perp$ is the vector potential of the electromagnetic pulse and ϕ the ambipolar potential due to charge separation in the plasma.

From equations (20) and (21) one finds that the perpendicular component of the electron momentum is

$$\frac{p_\perp}{m_ec} = \frac{e}{m_ec^2}\boldsymbol{A}_\perp \equiv \boldsymbol{a}(z,t). \tag{22}$$

Hence, we can write

$$\gamma = \left[1 + \left(\frac{p_\perp}{m_e c}\right)^2 + \left(\frac{p_z}{m_e c}\right)^2\right]^{1/2} \equiv \gamma_a \gamma_\parallel, \tag{23}$$

where

$$\gamma_a = (1 + \boldsymbol{a}^2)^{1/2}, \qquad \gamma_\parallel = (1 - \beta^2)^{-1/2} \tag{24}$$

and $\beta = v_z/c$.

The equations derived from this model are now the longitudinal component of equation (20), the electron continuity equation, Poisson's equation, and the wave equation for $\boldsymbol{a}(z, t)$, which are [49], respectively,

$$\frac{1}{c}\frac{\partial}{\partial t}\left(\gamma_a\sqrt{\gamma_\parallel^2 - 1}\right) + \frac{\partial}{\partial z}(\gamma_a\gamma_\parallel) = \frac{\partial\phi}{\partial z}, \qquad \varphi \equiv \frac{e\phi}{m_e c^2}, \tag{25}$$

$$\frac{1}{c}\frac{\partial n}{\partial t} + \frac{\partial}{\partial z}\left(n\frac{\sqrt{\gamma_\parallel^2 - 1}}{\gamma_\parallel}\right) = 0, \tag{26}$$

$$\frac{\partial^2\varphi}{\partial z^2} = \frac{\omega_{p0}^2}{c^2}\left(\frac{n}{n_0} - 1\right) \tag{27}$$

and

$$c^2\frac{\partial^2\boldsymbol{a}}{\partial z^2} - \frac{\partial^2\boldsymbol{a}}{\partial t^2} = \omega_{p0}^2\frac{n}{n_0}\frac{\boldsymbol{a}}{\gamma_a\gamma_\parallel}. \tag{28}$$

Assuming a driving pulse of the form

$$\boldsymbol{a}(z, t) = \tfrac{1}{2}\boldsymbol{a}_0(\xi, \tau)\exp(-\mathrm{i}\theta) + \text{c.c.}, \tag{29}$$

where $\theta = \omega_0 t - k_0 z$, ω_0 and k_0 being the central frequency and the wavenumber, respectively, $\xi = z - v_g t$, $v_g = \partial\omega_0/\partial k_0$ is the group velocity, and τ is a slow timescale such that

$$a_0^{-1}\frac{\partial^2 a_0}{\partial\tau^2} \ll \omega_0^2$$

and accounting for changes in the pump due to the plasma reaction, we obtain from equation (28)

$$\left[2\frac{\partial}{\partial\tau}\left(\mathrm{i}\omega_0 a_0 + v_g\frac{\partial a_0}{\partial\xi}\right) + c^2\left(1 - \frac{v_g^2}{c^2}\right)\frac{\partial^2 a_0}{\partial\xi^2} + 2\mathrm{i}\omega_0\left(\frac{c^2 k_0}{\omega_0} - v_g\right)\frac{\partial a_0}{\partial\xi}\right]\exp(-\mathrm{i}\theta)$$
$$+\text{c.c.} = \left[c^2k_0^2 - \omega_0^2 + \frac{n}{n_0}\omega_{p0}^2\gamma_a\gamma_\parallel\right]a_0\exp(-\mathrm{i}\theta) + \text{c.c.}, \tag{30}$$

where ω_{p0} is the plasma frequency of the unperturbed plasma, and c.c. stands for the complex conjugate. Equations (25)–(27) and (30) form the basic set for studying the laser wakefield excitation in the 'envelope approximation'.

In a weakly relativistic regime, the solution has the structure of a wakefield growing inside the electromagnetic pulse and oscillating behind the pulse with the maximum amplitude being reached inside the pulse. Using the quasi-static approximation the time derivative can be neglected in the electron fluid equations (25) and (26), yielding the following constants:

$$\gamma_a\left(\gamma_\parallel - \beta_0\sqrt{\gamma_\parallel^2 - 1}\right) - \varphi = 1 \tag{31}$$

and

$$n\big(\beta_0\gamma_\| - \sqrt{\gamma_\|^2 - 1}\big) = n_0\beta_0\gamma_\|, \tag{32}$$

where $\beta_0 = v_g/c$. The constants of integration have been chosen in such a way that

$$n = n_0, \qquad \gamma_\| = 1, \qquad \varphi = 0 \tag{33}$$

for $\gamma_a = 1$ and $|a_0|^2 = 0$.

Using equations (29) and (32) the general system of equations (25)–(28) can be written as two coupled equations [49] describing the evolution of the laser pulse envelope $\boldsymbol{a}_0$ and the scalar potential φ:

$$\frac{\partial^2\varphi}{\partial\xi^2} = \frac{\omega_{p0}^2}{c^2}G \tag{34}$$

and

$$2\mathrm{i}\omega_0\frac{\partial a_0}{\partial\tau} + 2c\beta_0\frac{\partial^2 a_0}{\partial\tau\,\partial\xi} + \frac{c^2\omega_{p0}^2}{\omega_0^2}\frac{\partial^2 a_0}{\partial\xi^2} = -\omega_{p0}^2 H a_0, \tag{35}$$

where

$$G = \frac{\sqrt{\gamma_\|^2 - 1}}{\beta_0\gamma_\| - \sqrt{\gamma_\|^2 - 1}} \qquad \text{and} \qquad H = 1 - \frac{\beta_0}{\gamma_a(\beta_0\gamma_\| - \sqrt{\gamma_\|^2 - 1})}.$$

The present set of nonlinear equations (34) and (35) are obtained by using a quasi-static approximation, which yields two integrals of motion given by equations (31) and (32). The model is valid for electromagnetic pulses of arbitrary polarization and intensities $|a_0|^2 \geqslant 1$.

Equations (34) and (35) can be solved numerically in the stationary frame of the pulse. Equation (34) is solved with the initial conditions $\varphi = 0$ and $\partial\varphi/\partial\xi = 0$ by a simple predictor–corrector method. The envelope equation (35), which describes the evolution of the laser pulse in the presence of the wake potential, is written as two coupled equations for the real and imaginary parts of a_0, and they can be solved implicitly. Numerical solutions of equations (34) and (35) show [49] the evolution of the excited plasma wakefield potential φ and the electric field E_w as well as the envelope of the laser pulse. There is a significant distortion of the trailing edge of the laser pulse resulting in photon spikes. The distortion occurs where the wake potential has a minimum and the density has a maximum. The spike arises as a result of the photons interacting with the plasma density inhomogeneity with some photons being accelerated (decelerated) as they propagate down (up) the density gradient; this effect, predicted again by John Dawson and his group [50] and by others [51], is called the photon accelerator. The distortion of the trailing edge increases with increasing ω_{p0}/ω_0. The longitudinal potential, namely $e\phi/mc^2 > 1$ or $eE_z/m_e\omega_{p0}c > 1$, is significantly larger than fields obtained in the PBWA, which are limited by relativistic de-tuning; no such saturation exists in the LWFA. Furthermore, Reitsma *et al* [42] proposed a new regime of laser wakefield acceleration of an injected electron bunch with strong bunch wakefields. In particular, the transverse bunch wakefield induces a strong self-focusing that reduces the transverse emittance growth arising from misalignment errors.

6. Self-modulated LWFA

Self-modulated LWFA is a hybrid scheme combining elements of SRFS and the laser wakefield concept. RFS describes the decay of a light wave at frequency ω_0 into light waves at frequencies

$\omega_0 \pm \omega_p$, and a plasma wave ω_p with $v_{\text{ph}} \simeq c$. Although RFS generates relativistic plasma waves and was identified as the instability that generated MeV electrons in early laser plasma experiments, it was not considered a serious accelerator concept because the growth rate is too small for sufficient plasma wave amplitudes to be reached before the ion dynamics disrupt the process. However, coupled with the LWFA concept it becomes a viable contender. Short pulse lasers have been demonstrated in [52, 61–63] to self-modulate in a few Rayleigh lengths. This modulation forms a train of pulses with an approximately $\pi c/\omega_p$ separation, which act as individual short pulses to drive the plasma wave. The process acts in a manner similar to that of a train of individual laser pulses. A comprehensive theoretical and simulation study of RFS and self-modulation pulses in tapered plasma channels have been presented by Penano *et al* [100].

A number of groups (e.g. [3–5, 44, 56–58]) have recently reported experimental evidence for the acceleration of electrons by relativistic plasma waves generated by a modulated laser pulse. The most impressive results come from a group working with the Vulcan laser at RAL, UK. This group, which consisted of research teams from Imperial College, UCLA, Ecole Polytechnique, and RAL, has reported observations of electrons at energies as high as 120 MeV [3–6]. The observations of energetic electrons was correlated with the simultaneous observation of $\omega_0 + n\omega_p$ radiation generated by RFS. The experiments were carried out using a 25 TW laser with intensities $> 10^{18}$ W cm^{-2} and pulse lengths < 1 ps in an underdense plasma $n_0 \sim 10^{19}$ cm^{-3}. The laser spectrum is strongly modulated by the interaction, showing sidebands at the plasma frequency [4]. Electrons with energies up to 100 MeV with an inferred minimum acceleration gradient of > 1.60 GV cm^{-1} over a measured 600 μm interaction length have been observed [4]. Laser self-channelling of up to 12 Rayleigh lengths was also observed in the experiment. Such extensive self-channelling was only observed for electron energies up to 40 MeV; at the higher energy $\sim$94 MeV the acceleration length was seven times shorter [4]. Similar results of a modulated laser pulse at ω_p have been obtained by a separate Livermore experiment [55] using a 5 TW laser but only observed 2 MeV electrons. The difference in the energy is explained by a difference in the length of plasma over which the acceleration takes place. In the RAL experiment the acceleration length is much larger. It is worth pointing out that in these experiments the accelerated electrons are not injected but are accelerated out of the background plasma. These experiments together with the short pulse wakefield have produced the largest—terrestrial acceleration gradients ever achieved—almost 200 GeV m^{-1}.

The RFS instability is the decay of an electromagnetic wave (ω_0, k_0) into a forward propagating plasma wave (ω_p, k_p) and forward propagating electromagnetic waves, the Stokes wave at $(\omega_0, -n\omega_p)$ and an anti-Stokes wave at $(\omega_0, +n\omega_p)$, where n is an integer; this is a four wave process. One of the earliest papers on forward Raman instabilities in connection with laser plasma accelerators was by Bingham [59] and McKinstrie *et al* [60] who discussed a purely temporal theory including a frequency mismatch. More recently a spatial temporal theory was developed in [61, 64]. In this theory, the relativistic plasma wave grows from noise; calculating the noise level is non-trivial since there are various mechanisms responsible for generating the noise. For example, the fastest growing Raman backscatter and sidescatter instabilities cause local pump depletion forming a ‘notch’ on the pulse envelope. The plasma wave associated with this notch acts as an effective noise source as seen in the simulations by Tzeng *et al* [65]. Simulations carried out by Tzeng *et al* [65] are the first to use the exact experimental parameters and show significant growth within a Rayleigh length. In fact, there is also a remarkable agreement with the experimental results of [3–5].

All these experiments rely heavily on extending the acceleration length, which is normally limited to the diffraction length or Rayleigh length $L_R = (\omega_0/2c)\sigma_0^2$, where ω_0 is the laser frequency and σ_0 the spot size. In present-day experiments this is limited to a few mm, for example, the RAL Vulcan CPA laser has a wavelength of 1 μm, a spot size of 20 μm resulting in a Rayleigh length $L_R \simeq 350\,\mu$m. To be a useful accelerator, laser pulses must propagate relatively stably through uniform plasmas over distances much larger than the Rayleigh length. Relativistic self-focusing is possible if the laser power exceeds the critical power given by $P_c \approx 17\omega_0^2/\omega_p^2$ GW, which is easily satisfied for the present high power laser experiments. There are difficulties in relying on the laser pulse to form its own channel since the beam may break up due to various laser-plasma instabilities, such as Raman scattering and filamentation. Relativistic self-guiding over five Rayleigh lengths has recently been reported in [66] using a 10 TW laser in a plasma of density 5×10^{18} cm^{-3}. They also note that the effect disappears at larger powers and densities. Alternatively, plasma channels have been demonstrated [67] to be very effective in channelling intense laser pulses over distances much larger than the Rayleigh length. In these experiments, a two laser pulse technique is used. The first pulse creates a breakdown spark in a gas target, and the expansion of the resulting hot plasma forms a channel that guides a second pulse injected into the channel. Pulses have been channelled up to 70 Rayleigh lengths [67], corresponding to 2.2 cm in the particular experiment with about 75% of the energy in the injected pulse focal spot coupled into the guide.

In a preformed channel other instabilities may appear. For example, it has been shown by Shvets and Wurtele [68] that a laser hose instability exists for parabolic channels. Plasma channels are not only important for laser plasma accelerators but have applications in high harmonic generation for UV and soft x-ray lasers.

However, the self-modulated LWFA mechanism would not work if the laser pulse length is of the order of the plasma wavelength. Nonetheless, one can still have electron trapping and acceleration as a result of the impulsive generation of plasma waves and its breaking [64, 70]. This process is known as the focused laser wakefield (FLWF) regime in which the laser pulse is compressed by group velocity dispersion. The leading edge of the compressed laser pulse can drive a relativistic plasma wave beyond its wavebreaking limit.

7. Particle beam driven PWFAs

Laser accelerator schemes are very effective for producing very high accelerating gradients over short plasma scales of the order of mm. Channelling coupled with guiding can increase this to several cm producing GeV's on a tabletop. Extending these tabletop accelerators needed for high energy physics requires metre scale plasmas; alternatively, high energies could be obtained by staging hundreds of these relatively compact laser accelerators, each providing gains of up to 10 GeV. Staging, however, is a formidable challenge requiring hundreds of lasers to be synchronized. Future lasers with intensities of the order of 10^{24} W cm^{-2} or greater created by optical parametric chirped pulse amplification may be powerful enough to propagate through metre long plasmas and produce ultra-high energy beams in the region 100 GeV–1 TeV suitable for high energy physics experiments. Another approach is to use the existing high energy particle beams that can also create wakefields in plasmas [28, 29]. A series of experiments at Stanford Linear Accelerator Center (SLAC) has demonstrated successfully the beam driven PWFA [24]. The SLAC beam is used in the experiments have intensities of the order of 10^{20} W cm^{-2}. In a particle-beam driven PWFA the Coulomb force due to the charged particle's space charge expels the plasma electrons if it is an electron beam and pulls them in if it is a positron beam. The displaced electrons snap back to restore charge neutrality and overshoot their original positions, setting up a plasma wave that trails behind the beam. If the beam is about half a plasma wavelength long, the plasma wave electric field amplitude in the linear theory, where $eE_p/M\omega_p c \ll 1$, is given by [23]

$eE_p = 240\,(\text{MeV m}^{-1})(N/4 \times 10^{10})(0.6\,\text{mm}/\sigma_z)^2$, where σ_z is the bunch length and N is the number of particles per bunch. The driver of the plasma wakefield experiment in the SLAC experiment was the 28.5 GeV, 2.4 ps long positron beam from the SLAC linac containing 1.2×10^{10} particles. The beam was focused at the entrance to a 1.4 m long chamber filled with a lithium plasma of density $1.8 \times 10^{14}\,\text{cm}^{-3}$. The first two-third of the positron beam sets up by the relativistic plasma wakefields while the last third of the beam was accelerated to higher energy. The main body of the beam decelerated at a rate corresponding to 49 MeV m^{-1}, while the back of the beam containing about 5×10^8 positrons in a 1 ps slice was accelerated by 79 ± 15 MeV in the 1.4 m long plasma corresponding to an accelerating electric field of 56 MeV m^{-1}. The results are in excellent agreement with a three-dimensional particle-in-cell (PIC) simulation [24], which predicted a peak energy gain of 78 MeV. In the beam driven wakefield experiment the beam energy is transferred from a large number of particles in the core of the bunch to a fewer number of particles at the back of the same bunch. The wakefield thus acts like a transformer with a ratio of accelerating field to decelerating field of about 1.3 in the experiment. This is the first demonstration that plasma accelerators can be used to increase the energy of the high energy accelerator and leads the way for a possible energy doubler proposed by Lee *et al* [22]. In the energy doubler or plasma ‘afterburner’ scheme, a PWFA using ten times shorter bunches can double the energy of a linear collider in short plasma sections of length 10 m. By reducing the positron bunch by a factor of 10 the accelerating gradient can be enhanced a hundredfold to 5 GV m^{-1}, which would convert the 50 GeV SLAC linear collider into a 100 GeV machine [22]. To sustain the luminosity at the interaction point at the nominal level of the original beam without the plasma, the reduction in the number of higher energy particles is compensated by using plasma lenses to reduce the spot size. Higher density plasma lenses are added to the afterburner design just before the interaction point. This type of plasma lens exceeds conventional magnetic lenses by several orders of magnitude in focusing gradient and was successfully demonstrated [71]. Wakes generated by an electron or positron beam can also accelerate positrons and muons. These SLAC experiments have brought the concept of plasma accelerators to the forefront of high energy accelerator research.

8. Photon acceleration

Photon acceleration [72, 51] is intimately related to short pulse amplification of plasma waves and is described by a set of equations that are similar to those in wakefield accelerators. In the last section on wakefield amplification we noted that it was much easier to increase the amplitude of the wakefield if we used a series of photon pulses. These pulses have to be spaced in a precise manner to give the plasma wave the optimum ‘kick’. If the second or subsequent photon pulses are put in a different position, 1.5 plasma wavelengths behind the first pulse, the second pulse will produce a wake that is 180° out of phase with the wake produced by the first pulse. The superposition of the two wakes behind the second pulse results in a lowering of the amplitude of the plasma wave. In fact, almost complete cancellation can take place.

The second laser pulse has absorbed some or all of the energy stored in the wake created by the first pulse. Conservation of photon number density implies that an increase in energy of the second pulse means an increase in its frequency. The energy in the pulse is $W = N\hbar\omega$, where N is the total number of photons in the wave packet.

A simple and rigorous account of the frequency shift can be given by using the photon equations of motion [73]:

$$\frac{\mathrm{d}\boldsymbol{r}}{\mathrm{d}t} = \frac{\partial\omega}{\partial\boldsymbol{k}}, \qquad \frac{\mathrm{d}\boldsymbol{k}}{\mathrm{d}t} = -\frac{\partial\omega}{\partial\boldsymbol{r}}, \tag{36}$$

where $\boldsymbol{r}$ and $\boldsymbol{k}$ are the photon position and the wavevector, respectively, and $\omega = (k^2c^2 + \omega_p^2(\boldsymbol{r}, t))^{1/2}$ is the photon frequency.

A net frequency shift (or photon acceleration) occurs in non-stationary plasmas as well as in other time varying media. Let us take, as an example, the case of a laser (or photon) beam interacting with a relativistic ionization front with the velocity v_f. From equation (36) one can readily obtain the net frequency shift of the laser beam

$$\Delta\omega = \frac{\omega_{\mathrm{pe}}^2}{2\omega}\frac{\beta}{1 \pm \beta}, \tag{37}$$

where $\beta = v_f/c$, and the plus and minus signs are for co- and counter-propagation of the photons, respectively, with respect to the ionization front. This was confirmed by experiments [74]. Quite recently, frequency shifts as large as 144 nm were observed for ionization fronts with a velocity of $0.99c$ [75]. Experiments designed to observe photon acceleration by laser produced wakefields are currently being prepared at RAL in collaboration with IST and Imperial College Groups.

9. Relativistic self-focusing and optical guiding

In the absence of optical guiding the interaction length L is limited by diffraction to $L = \pi R$, where $R = \pi\sigma^2/\lambda_0$ is the Rayleigh length and σ is the focal spot radius. This limits the overall electron energy gain in a single stage to $E_{\mathrm{max}}\pi R$. To increase the maximum electron energy gain per stage, it is necessary to increase the interaction length. Two approaches to keeping high energy laser beams collimated over longer regions of plasma are being developed. Relativistic self-focusing can overcome diffraction and the laser pulse can be optically guided by tailoring the plasma density profile forming a plasma channel [76–79]. Guiding of high intensity laser pulses in straight and curved plasma channel experiments has been demonstrated by Ehrlich *et al* [80].

Relativistic self-focusing uses the nonlinear interaction of the laser pulse and plasma resulting in an intensity dependent refractive index to overcome diffraction [81]. In regions where the laser intensity is highest, the relativistic mass increase is greatest; this results in a reduction of the fundamental frequency of the laser pulse. The reduction is proportional to the laser intensity. Correspondingly, the phase speed of the laser pulse will decrease in regions of higher laser intensity. This has the effect of focusing a laser beam with a radial Gaussian profile. This results in the bending of the plane wavefront and its focusing to a smaller spot size. Relativistic self-focusing has a critical laser power threshold, P_{cr}, given by

$$P \geqslant P_{\mathrm{cr}} \simeq \frac{2m_e^2c^5\omega_0^2}{e^2\omega_p^2} = 17\frac{\omega_0^2}{\omega_p^2} \quad (\mathrm{GW}). \tag{38}$$

The laser must also have a pulse duration that is shorter than both the collisional and ion plasma periods to avoid the competing effects of the thermal and ponderomotive self-focusing. The shape of the self-focused intense short pulses is also interesting. There is a finite time for the electrons to respond and the front of the pulse propagates unchanged. The trailing edge of the pulse compresses radially due to the nonlinear relativistic self-focusing. It is only the trailing edge of the pulse that is channelled. Several authors [82–84] have presented analytical studies of self-focusing and relativistic self-guiding of an ultrashort laser pulse in the plasma, taking into account the ponderomotive force and relativistic mass variation nonlinearities. The ponderomotive force of superstrong electromagnetic fields expels electrons, thus producing ‘vacuum channels’ that guide the radiation and stable channelling with power higher than the critical one can take place. Specifically, Cattani *et al* [84] presented a simple model to investigate multifilament structures of a circularly polarized laser beam in two-dimensional planar geometry, including electron cavitation, which is created by the relativistic ponderomotive force. The governing equations describing filamentary structures are [84]

$$\nabla^2 a_\perp + \left(1 - \frac{\alpha n}{\gamma_\perp}\right) a_\perp = 0 \tag{39}$$

and

$$\nabla^2 \varphi = \alpha(n-1), \tag{40}$$

where $\varphi = \gamma_\perp - 1$ if and only if $n \neq 0$, $\gamma_\perp = \sqrt{1 + a_\perp^2}$ and $\alpha = N_0/(1 - k_z^2/k^2)$. Here, we have denoted $a_\perp = e|\boldsymbol{A}_\perp|/m_e c^2$, $N_0 = n_0/n_c$, $n_c = m_e \omega^2/4\pi e^2$, $\varphi = e\phi/m_e c^2$, and $\boldsymbol{r} = k\sqrt{1 - k_z^2/k^2}\boldsymbol{r}_\perp$, with $k_\perp \ll k_z$. Here, $k_\perp$ is the transverse component of the laser wave number and k_z is the axial propagation constant. Equations (38) and (39) admit an exact soliton-like solution as well multifilament structures whose specific forms are presented by Cattani *et al* [84].

Another way of constructing a guided laser is to create a preformed plasma with one long pulse laser, which heats and expands on a ns timescale forming a plasma density channel [85–87]. Alternatively, long plasma channels could also be formed by high voltage capillary discharges, which produce the same effect [88,89]. Bulanov *et al* [90] have presented the results of a two-dimensional PIC simulation of the nonlinear propagation of a short, relativistically intense laser pulse in an underdense plasma. They found that such a pulse can be focused in a plasma with strong magnification of its amplitude and channelling in a narrow channel shaped like a 'bullet'. Sprangle *et al* [91] have studied the dynamics of short laser pulses propagating in plasma channels, taking finite-pulse-length as well as nonlinear focusing effects into account. They found that, in plasma channels, pulses can undergo an envelope modulation, which is damped in the front and initially grows at the back of the pulse. Finite-pulse-length effects also significantly increase the nonlinear focusing. A nonlinear theory for non-paraxial laser pulse propagation in plasma channels is presented by Esarey *et al* [92] who, in the adiabatic limit, analysed pulse energy conservation, nonlinear group velocity, damped betatron oscillations, self-steepening, self-phase modulation, and shock formation. In the non-adiabatic limit, the nonlinear coupling of FRS and the self-modulational instability leads to a reduced growth rate. The possibility of stable laser pulse in a tapered plasma channel for GeV electron acceleration has also been discussed [93], including wakefields and relativistic and non-paraxial effects.

Recently, it has been shown [94] that the relativistic self-focusing in three dimensions of a linearly polarized laser pulse propagating in an underdense plasma with power above the threshold power P_{cr} is anisotropic and evolves differently in the plane in which the pulse electric field oscillates and in the plane in which the magnetic field oscillates. The effect of the pulse polarization and of the accelerated fast electrons on the propagation anomalies ('hosing' [95] and 'snaking') of a high-intensity laser pulse has been investigated [96] by means of fully electromagnetic relativistic two-dimensional PIC simulations. The results show that hosing [97, 98] occurs in the front of the pulse, and it corresponds to low-frequency oscillations of the electron current at the sharp front of the pulse and to a periodic change of the refractive index. The hosing of the pulse may be responsible for self-trapped electrons as well as for the excitation of 'wake surface waves' [99] at the walls of the self-focusing channel. Snaking occurs in the later evolution of long laser pulses almost independently of their polarization.

In these laser experiments it is, in principle, feasible to reach 1 GeV range of energies without resorting to hybrid mechanisms. In the future, in order to achieve TeV energies substantial research must be carried out with all combinations, larger plasma channels should be developed and staging should be introduced. These experiments will make it possible to judge whether a TeV machine is possible with desirable luminosities and emittances. Future high energy accelerators may combine conventional accelerators together with a plasma afterburner to increase the beam energy. The SLAC experiments suggest that the beam can be doubled in energy if the pulse length is halved.

10. Conclusions and outlook

Plasma acceleration processes continue to be an area of active research. The initial studies of particle acceleration have provided schemes fruitful for current drive and laser accelerators. Particle acceleration in strongly turbulent plasmas is still in its infancy and requires a great deal more research. This area of research is important in astrophysical and space plasmas as well as in high energy physics.

The present and future laser experiments, however, are very far from the parameter range of interest to high energy physicists, who require something like 10^{11} particles per pulse accelerated to TeV energies (for electrons) with a luminosity of 10^{34} erg cm^{-2} s^{-1} for acceptable event rates to be achieved. The TeV energy range is more than 1000 times greater than that of a single laser accelerating stage could provide at present; even if the interaction length can be extended by laser channelling the requirement of multiple staging and more energetic lasers has to be met. A TeV beam of 10^{11} particles per pulse and a transfer efficiency of 50% would require a total of 32 kJ of laser energy per pulse for a 100 stage accelerator. This would require 100 lasers of about 300 J each with high repetition rates in contrast to the 56 J lasers in the proposed GeV accelerator and the 1 J laser used in present-day experiments (10^{10} particles per pulse would require 100×30 J lasers). Beam driven plasma wakefields that contain conventional accelerators with plasma accelerators may be the way forward in the near term, while more intense lasers are developed using the optical parametric CPA process.

The work on plasma-based accelerators represents one area that is being explored by researchers in the advanced accelerator field. Other schemes being investigated at present for high-gradient acceleration are the inverse Cherenkov effect and the inverse free-electron laser (FEL) effect. Still other researchers, realizing that the next collider will almost certainly be a linear electron–positron collider, are proposing a novel way of building such a device known as a two-beam accelerator, and there are many groups developing an entirely new type of electron lens using focusing by a plasma to increase the luminosity of future linear colliders [71, 101]. This makes use of the fact that relativistic electron beams can be focused by a plasma if the collisionless skin depth $c/\omega_{\rm pe}$ is larger than the beam radius. Generally, when a relativistic electron beam enters a plasma, the plasma electrons move to neutralize the charge in the beam on the electron plasma period timescale. However, if the skin depth is larger than the beam radius, the axial return current flows in the plasma on the outside of the electron beam and the beam current is not fully neutralized, leading to the generation of an azimuthal magnetic field. Consequently, self-generated magnetic fields pinch or focus the beam in the radial direction. This type of plasma lens [102] exceeds conventional lenses by several orders of magnitude in focusing gradient. Currently, an experiment is underway at the SLAC to demonstrate wakefield excitation by an electron bunch. Clayton *et al* [103] studied experimentally the transverse dynamics of a 28.5 GeV electron beam in a 1.4 m long underdense plasma (with $n_0 \sim 2 \times 10^{14}$ cm^{-3}). The transverse component of the wakefield excited by the short electron bunch focuses the bunch, which experiences multiple betatron oscillations. One is thus developing new technologies to reduce the size and cost of future elementary particle physics experiments based on colliding high energy beams, even including positron beams. Lee *et al* [22] have presented three-dimensional PIC simulations and physical models for plasma-wakefield excitation by positron beams. They found that the nonlinear wake of a positron bunch is smaller than that of an electron bunch, but it can be made comparable to the electron wake by employing a hollow plasma.

For laser plasma accelerators, the next milestone to be achieved is 100 MeV–1 GeV energy levels with good beam quality. Electrons in this energy range are ideal as a driver for FELs. At higher energies, of several GeV, it is possible to produce an x-ray FEL capable of biological investigations around the water window. Furthermore, a high intensity laser pulse interacting with a plasma can produce intense proton beams that can be used for treatment of oncological diseases [104]. Yamagiwa *et al* [105] have carried out two-dimensional PIC simulations to show that the longitudinal electric field induced by electron evacuation due to the intense light pressure can accelerate ions to several MeV in the direction of the laser propagation.

The work on plasma accelerators by high power lasers has applications in astrophysics in that one is able to explain the origin of energetic particles from compact highly bright objects. Raman scattering has already been discussed in the astrophysics literature, in particular the eclipsing pulsar radio sources [106]. Forward Raman scattering, which has not been considered in this area, will produce relativistic plasma waves, which have the capability of accelerating particles to energies greater than TeV energies. For example, forward Raman scattering in a plasma of density 10^7–10^8 cm^{-3} is capable of generating plasma wave turbulence with mean electric field values of $\approx$1 V cm^{-1}. The quasi-linear diffusion of particles in such fields over distances of 10^{15}–10^{16} cm would result in such an energetic particle, thus avoiding the need for the Fermi acceleration. It is envisaged that particle acceleration by relativistic plasma waves in JETs could sustain the acceleration process over hundreds of parsecs. Particles that lose energy through radiation losses are re-accelerated within the JET. The source of relativistic plasma waves are the relativistic particles themselves. Relativistic electrons generate the relativistic plasma wave in a manner similar to the PWFA. Recently, Chen *et al* [107] suggested that the plasma wakefield mechanism could be responsible for ultra-high-energy cosmic rays exceeding the Greisen–Zatsepin–Kuzmin (GZK) limit.

An exciting development for high powered lasers with ultra-high intensity ($>10^{23}$ W cm^{-2}) ranges is the study of the Unruh radiation [108, 109], which requires very large acceleration to be detectable. This radiation has been likened to Hawking radiation [110] from black holes. The weak decay of uniformly accelerated protons in the context of standard quantum field theory has been recently investigated by Vanzella and Matsas [111]. Experiments in this area can be carried out using high powered lasers in ionizing a gas rapidly, changing the refractive index within one cycle of the laser. Such rapid refractive index changes can produce a reference frame accelerating with $\gtrsim 10^{20} g$, where g is the acceleration on Earth. These experiments are associated with vacuum energy, which in the future could be harnessed to accelerate particles to very high energies, greater than that available in present-day experiments. The ponderomotive force of a short laser pulse can generate ultra-bright attosecond electron bunches [114] in vacuum. The laser-pulse profile can be tailored in such a manner that electrons are both focused and accelerated by the light pressure. Furthermore, there also exists the possibility of accelerating electrons in the laser electromagnetic field, to energies exceeding, by an order of magnitude, their oscillation energy [112]. Pukov [113] has presented a comprehensive review of electron acceleration in the wake-wave field and electron betatron resonance in quasi-static electric and magnetic fields of a plasma channel. In plasmas with the transverse density gradient $\partial n_0/\partial r$, a recent experiment [115] reveals the emission of a second-harmonic due to the spatially asymmetric relativistically quivering electron motion produced by the ponderomotive force of an intense laser pulse. The power of the second-harmonic emission is [115]

$$P_{2\omega} \propto \frac{a_{\perp 0}^4}{1 + a_{\perp 0}^2/2} \left(\frac{n_0}{L_n}\right)^2, \tag{41}$$

where $a_{\perp 0} = e|\boldsymbol{A}_{\perp 0}|/m_e c^2$ corresponds to the intensity of the pump pulse and $L_n = n_0(\partial n_0/\partial r)^{-1/2}$ is the density gradient scalelength, which is largest at the beam edges.

Finally, we mention that short intense laser pulses can spontaneously create megagauss magnetic fields [116, 117], which affect the dynamics of electrons in plasmas. Consequently, both the relativistic ponderomotive force and wakefields are influenced [118–120] by the presence of these magnetic fields. The surfatron acceleration mechanism [121] can produce unlimited electron acceleration due to the cross wave-electric and external magnetic fields. In fact, Ucer and Shapiro [122] and McClements *et al* [123] have exploited the idea of the surfatron mechanism for unlimited relativistic shock surfing acceleration of ions and acceleration of cosmic ray electrons by waves excited by ions reflected from supernova remnant (SNR) shocks, respectively. In the presence of ultrastrong magnetic fields, resonance electrons can continuously gain energy from the circularly polarized laser field [124]. Furthermore, in a magnetized plasma we also have the possibility of radiation generation from the Cherenkov wake excited by ultrashort intense laser pulses. Recently, Yugami *et al* [125] have presented a proof-of-principle experiment demonstrating the emission of radiation in the mm range (up to 200 GHz). The intensity of the radiation is proportional to the magnetic field strength. On the other hand, Yu *et al* [126] have considered electron acceleration and high-harmonic (short-wavelength radiation) generation by an intense short linearly polarized laser pulse in an external magnetic field. They found that electrons can be strongly energized, and most of the energy gained by electrons is retained in its relativistic cyclotron motion, even though the pulse has disappeared. The energetic electrons, in turn, emit radiation at high harmonics of the cyclotron frequency. In view of the above mentioned recent studies, it is suggestive that future research should concentrate on the formation of three-dimensional wakefields and subsequent particle acceleration as well as on related phenomena (e.g. self-focusing and optical guiding, generation of small scale density and magnetic field perturbations [127, 128], etc) by including the dc magnetic field and the background plasma non-uniformity. It should be stressed that the generation of wakefields by relativistic electron or positron bunches is certainly one of the potential candidates for high energy electron acceleration in existing PWFA experiments. Nonlinear interactions between a short, intense laser pulse and a plasma can also be used to accelerate ions [129] and to generate positrons [130]. Furthermore, computer simulations [131–133] have been performed to demonstrate the generation of multi-MeV electrons by direct laser acceleration, a topic which we did not touch upon in this review. There are other schemes for electron acceleration by lasers in vacuum, gases and plasmas [134]. Some new ideas for two laser accelerators [135] and an x-ray FEL [136], based on high energy electron accelerators (with energies exceeding 10 GeV), have been proposed. We, therefore, believe that the plasma based charged particle accelerators at the energy frontier and on tabletops will definitely provide breakthroughs in our understanding of the physics and technology of charged particle acceleration, as envisioned in [137].

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