

**Department of Physics**

**PH384 Quantum Physics and Electromagnetism**

**April 19th 2024** **14:00 – 17:00 - 3 hours**

**Attempt ALL questions in Section A (40%) AND in Section B (60%)**

***Calculators must not be used to store text and/or formulae nor be capable of communication. Invigilators may require calculator resets.***

Physical constants

Speed of light  = 3.00×108 m s-1 Bohr radius = 5.29×10-11 m

Electric constant = 8.85×10-12 F m-1 Compton wavelength = 2.43×10-12 m

Magnetic constant = 1.26×10-6 H m-1 Avogadro’s number = 6.02×1023 mol-1

Electrostatic constant = 8.99×109 m F-1 Boltzmann’s constant = 1.38×10-23 J K-1

Electron mass = 9.11×10-31 kg Universal gas const. = 8.31 J mol-1 K-1

Electron charge size = 1.60×10-19 C Gravity acc. on earth = 9.81 m s-2

Planck’s constant = 6.63×10-34 J s Gravitational const. = 6.67×10-11 N m2 kg-2

Wien’s law constant 2.90×10-3 m K Stefan-Boltzmann = 5.67×10-8 W m-2 K-4

Rydberg constant = 1.10×107 m-1 Atomic mass unit = 1.66×10-27 kg

**Section A (attempt ALL questions)**

**Q1 a)** The *time-dependent* Schrödinger equation is in one dimension. Identify the energies associated with each term in the equation. **[2]**

**b)** By substituting into the *time-dependent* Schrödinger equation from **a)** a solution that is separable in space and time, i.e. , where function , rearrange and simplify to arrive at the time-independent Schrödinger equation.

**[6]**

**c)** What is the meaning of the constant ? What assumption was needed about the potential energy’s dependence on time and/or space in **b)**?

**[2]**

**Q2** A hydrogen atom is repeatedly prepared in the *same* quantum orbital angular momentum superposition state , where the refer to the usual states from the orthonormal basis set.

**a)** What condition must apply to the complex constant in order to normalise ? **[2]**

**b)** In repeated measurements of the square of the orbital angular momentum, what are the two possible outcomes of this measurement? **[2]**

**c)** In repeated measurements of the component of the orbital angular momentum, what are the two possible outcomes of this measurement? **[2]**

**d)** What are the expectation values and ? **[4]**

**Q3.** Maxwell’s equations can be written as

where is the electric field, is the magnetic field, is the electric charge density, is the electric current density, is the electric constant and is the magnetic constant.

1. If magnetic monopoles were discovered tomorrow, the Faraday-Lenz law and which other Maxwell equation would need to be modified? **[1]**
2. Considering the electromagnetic field in *vacuum*, use Maxwell’s equations above to derive one of the following electromagnetic wave equations (the choice of which is up to you):

In doing so, give an explicit expression for speed of light in terms of and . You can use any of the vector identities listed on the formula sheet without proof. Hint: start by writing down expressions for and in vacuum, then take the curl of both sides of one of Maxwell’s equations. **[8]**

1. Over what frequency range of the electromagnetic spectrum do the wave equations in part **b)** above apply? **[1]**

**Q4** The propagation of a monochromatic electromagnetic wave in a conductor can be described using a complex refractive index

whereis the permittivity of the conductor, is the permeability of the conductor, is the conductivity of the conductor and is the angular frequency of the wave.

1. Show that in a *good* conductor, can be well approximated by

**[2]**

1. Consider an electromagnetic wave of frequency propagating in silver. Calculate the phase speed of the wave, treating silver as a non-magnetic, good conductor with conductivity . Hint: you can extract an expression for the refractive index of a good conductor from part **a)** above. **[4]**
2. If the electric-field amplitude of the wave is at , what is the electric field amplitude at , where is the skin depth of the conductor. **[2]**
3. The wave loses energy as it propagates through the conductor. Is this a violation of the conservation of energy? Briefly explain your answer. **[2]**

**Section B (attempt ALL questions)**

**Q5** Two identical particles at positions and have spatial wavefunctions and , respectively.

1. If the probability density is to be invariant under particle exchange, write down the two possible wavefunctions of the two-particle system. **[2]**
2. Show that one of these wavefunctions is enhanced and the other vanishes for two particles in the same position. **[2]**
3. What are the names given to particles with wavefunctions that satisfy different symmetry properties upon exchange of positions? **[2]**

**Q6** Consider a one-dimensional quantum harmonic oscillator of mass , angular frequency , and Hamiltonian with normalised eigenfunctions that satisfy the Schrodinger equation .

1. Show the Hamiltonian can be written in terms of the raising and lowering operators

, as **either**

**or** **[4]**

1. Verify the commutator . **[3]**
2. If the action of the raising and lowering operators on the energy eigenfunctions is as follows (for all *n*) and (= 0 for *n* = 0), use the first part of the expression for the Hamiltonian in **a)** in the Schrodinger equation to show that the energy eigenvalues are . **[8]**

**Q7** For a spin-1/2 particle the Pauli spin matrices representing total spin and the amount of spin in the -direction, -direction, and -direction respectively are:

1. Calculate the matrix representing the commutator . **[3]**
2. How and why should your answer for **(i)** be related to the operator ? **[2]**
3. From the properties of the commutators (or otherwise) expand and express in terms of spin operators. **[4]**

**Q8** The electromagnetic energy density and Poynting’s vector are given by

1. Show that

Hint: take the time derivative of both sides of the definition above for then use two of Maxwell’s equations.

**[5]**

1. Use a vector identity to show that the expression given in part **a)** above can be cast in the form

In doing so, give an expression for the Lorentz power density in terms of and . You can use any of the vector identities listed on the formula sheet without proof.

**[3]**

1. What is the physical significance of the differential equation shown in part **b)** above?

**[2]**

Consider now a monochromatic electromagnetic wave propagating in vacuum with complex fields

1. Show by explicit calculation that

Hint: you will need to extract expressions for the (real) fields and from the expressions above for and .

**[3]**

1. Use your answer to part **d)** above to derive an expression for the intensity of the wave in terms of , and .

**[2]**

**Q9** The Fresnel *amplitude* reflection and transmission coefficients for a boundary separating two non-magnetic dielectrics are

with , and , where and are the refractive indices of the dielectrics, is the angle of incidence and is the angle of refraction.

Consider light incident at an angle of on an air-glass boundary with its polarisation *perpendicular* to the plane of incidence. Take for air and for glass.

1. Use Snell’s law to show that the angle of refraction is . **[2]**

1. Calculate the ratio of the *intensity* of the reflected light to the *intensity*of the incident light.  **[4]**
2. Calculate the ratio of the *intensity* of the transmitted light to the *intensity*of the incident light.  **[4]**
3. A colleague claims to have developed a machine that can directly measure a scalar potential and vector potential . Briefly comment on their claim - is it valid? **[2]**

In a certain gauge, Maxwell’s equations are solved mathematically by the *advanced* potentials

where is the *advanced* time.

1. Name the gauge used. **[1]**
2. Briefly explain why the advanced potentials are usually rejected as being unphysical. **[2]**

**END OF PAPER**

(Dr A. S. Arnold and Dr. R. P. Cameron)