Charge exchange cross sections for B^{5+} and other fully stripped ions with H and applications.

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Outline

1. Motivation
2. Theoretical Methods
3. B^{5+}+ H Collision System
4. Other fully stripped ions and CRS diagnostic

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Motivation

- Impurity concentration afects negatively the fusion power density.
- CXRS is used as plasma diagnostic for Ti and density.
- Very accurate cross sections are required to adequately model the impurity density in plasmas.
- Using different methods we can give cross sections data in a wide range of energies.

Different methods for B^{5+} + H calculation

Calculations peformed

$\swarrow$ $\downarrow$ $\searrow$

	Quantal	Semiclassical	Classical
capture	Yes	Yes	Yes
ionization	No(Yes*)	No(Yes*)	Yes
excitation	No(Yes)	Yes	No(Yes)
Energy interval (keV/amu)			
$\mathbf{B^{5+}}$+H(1s)	$0.01 \leq E \leq 1$	$0.25 \leq E \lesssim 28.58$	$35.97 \lesssim E \leq 1000$
B^{5+} +H(2s)	$0.01 \leq E \leq 1$	$0.25 \leq E \lesssim 15.41$	$19.50 \lesssim E \leq 1000$

*: incluing pseudostates

Molecular Quantal Method

Common Reaction coordinate

Molecular Quantal Method

Common Reaction coordinate

- Electronic and nuclear motion are described by quantum mechanics.

$$H\Psi = E\Psi \quad \begin{cases} \Psi(\mathbf{r},\boldsymbol{\xi}) & \xrightarrow[\xi\to\infty]{} \quad \phi_i^A e^{i\mathbf{k}_i'\boldsymbol{\xi}} + \sum_f \frac{e^{ik_f'\xi}}{\xi} f_{if}'(\Theta)\phi_f^A \\ \Psi(\boldsymbol{r},\boldsymbol{\xi}) & \xrightarrow[\xi\to\infty]{} \quad \sum_f \frac{e^{ik_f'\xi}}{\xi} f_{if}'(\Theta)\phi_f^B \end{cases}$$

Molecular Quantal Method

Common Reaction coordinate

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$$H\Psi = E\Psi$$

$$\begin{aligned} \xi(\boldsymbol{r},\boldsymbol{R}) &= \boldsymbol{R} + \frac{1}{\mu}\boldsymbol{s}(\boldsymbol{r},\boldsymbol{R}) \\ \boldsymbol{s}(\boldsymbol{r},\boldsymbol{R}) &= f(\boldsymbol{r},\boldsymbol{R})\boldsymbol{r} - \frac{1}{2}f^2(\boldsymbol{r},\boldsymbol{R})\boldsymbol{R} \end{aligned}$$

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$$\Psi(\boldsymbol{r},\boldsymbol{\xi}) = \sum_J \Psi^J(\boldsymbol{r},\boldsymbol{\xi}) = \sum_J \sum_k \chi_k^J(\boldsymbol{\xi})\Phi_k(\boldsymbol{r},\xi)$$

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- $\{\Phi_k\}$ are Born-Oppenheimer eigenfunctions for R=ξ.

$$H_{elec}(\boldsymbol{r},\xi)\Phi_k(\boldsymbol{r},\xi) = E_k\Phi_k(\boldsymbol{r},\xi)$$

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- Cross Section to the state j from the initial state i

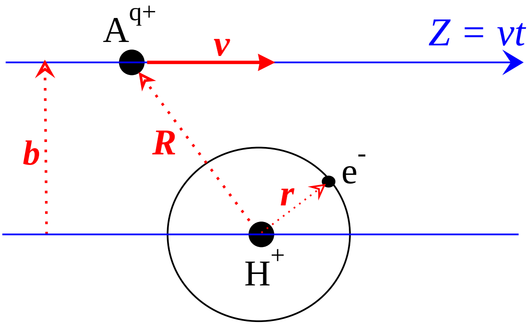
$$\boxed{\sigma_{ij} = \frac{\pi}{k_i^2}\sum_J (2J+1)|\delta_{ij} - S_{ij}^J|^2}$$

Semiclassical Method

Eikonal approach

At big impact energies ($E > 250eV/uma$) nucleai motion can be approach by straight trajectories:

$$\boldsymbol{R}(t) = \boldsymbol{b} + \boldsymbol{v}\, t$$



Semiclassical Method

Eikonal ecuation

Electronic motion is described by $\Psi(\boldsymbol{r};t)$ that is solution of the eikonal equation:

$$i\left(\left.\frac{\partial\Psi(\boldsymbol{r};t)}{\partial t}\right|_{r}\right) = H_{el}\Psi(\boldsymbol{r};t)$$

$\Psi(\boldsymbol{r};t)$ is expanded in molecular orbitals (exact, variacional):

$$\Psi(\boldsymbol{r},t) = \mathrm{e}^{iU(\boldsymbol{r},R)}\sum_{j}^{N} a_j(t)\Phi_j(\boldsymbol{r};R)\exp\left[-i\int^{t} E_j(t')dt'\right]$$

with U=CTF.

Semiclassical Method

Cross Sections

Coupled equation system:

$$\begin{aligned}\frac{\mathrm{d}a_k(t)}{\mathrm{d}t} &= \sum_j a_j(t)\Bigg(\left\langle \Phi_k \left| H_{el} - \mathrm{i}\frac{\partial}{\partial t}\right| \Phi_j \right\rangle + \left\langle \Phi_k \left| \frac{1}{2}(\nabla U)^2 + \frac{\partial U}{\partial t} \right| \Phi_j \right\rangle + \\ &- \mathrm{i}\left\langle \Phi_k \left| -\frac{1}{2}\nabla^2 U - \nabla U \cdot \nabla \right| \Phi_j \right\rangle\Bigg) \exp\left[-i\int_0^t (E_j(t') - E_k(t'))\mathrm{d}t'\right]\end{aligned}$$

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$$\left. - \quad \mathrm{i}\left\langle \Phi_k \middle| -\frac{1}{2}\nabla^2 U - \nabla U \cdot \nabla \middle| \Phi_j \right\rangle \right) \exp\left[-i\int_0^t (E_j(t') - E_k(t'))\mathrm{d}t'\right]$$

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Cross Sections:

$$\boxed{\sigma_{nlm}^{A,B}(v) = 2\pi \int |a_{nlm}^{A,B}(v, b, t \to \infty)|^2 b\mathrm{d}b.}$$

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$$|S_{ij}|^2 = P_{ij}(b) = |a(v, b, t \to \infty)|^2$$

Classical CTMC Method

$E > 25$keV/amu

Electronic motion is described by a statistical distribution of N punctual charges that do not interact:

$$\rho(\boldsymbol{r},\boldsymbol{p},t) = \frac{1}{N}\sum_{j=1}^{N}\delta\left(\boldsymbol{r}-\boldsymbol{r}_j(t)\right)\delta\left(\boldsymbol{p}-\boldsymbol{p}_j(t)\right)$$

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$\downarrow$ Liouville Equation:

$$\frac{\partial \rho}{\partial t} = -\left\{\rho, H_{el}\right\} = -\frac{\partial \rho}{\partial r} \cdot \frac{\partial H_{el}}{\partial p} + \frac{\partial \rho}{\partial p} \cdot \frac{\partial H_{el}}{\partial r}$$

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Obtainig the Hamilton Equations:

$$\left.\begin{aligned} \dot{r}_j(t) &= \frac{\partial H}{\partial p_j(t)} \\ \dot{p}_j(t) &= -\frac{\partial H}{\partial r_j(t)} \end{aligned}\right\}$$

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$$P_{c,e,i}(v,b) = \int d\boldsymbol{r}\int d\boldsymbol{p}\,\rho_{c,e,i}(\boldsymbol{r},\boldsymbol{p},t_{max}) = \frac{N_{c,e,i}}{N_{\mathrm{Total}}}$$

$$\boxed{\sigma_{c,e,i}(v) = 2\pi\int_0^{\infty} db\, b\, P_{c,e,i}(v,b)}$$

Classical CTMC Method

Initial Conditions

Initial Distributions $\rho(\boldsymbol{r}, \boldsymbol{p}, t \to -\infty)$:

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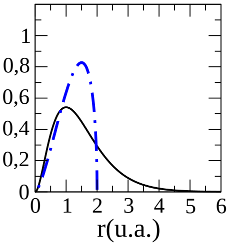
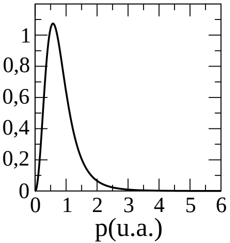
- Microcanonical Distribution:

$$\rho^m(\boldsymbol{r}, \boldsymbol{p}; E_0) = \frac{(2|E_0|)^{5/2}}{8\pi^3 Z_H^3} \delta\left(\frac{p^2}{2} - \frac{Z_H}{r} - E_0\right)$$





Classical CTMC Method

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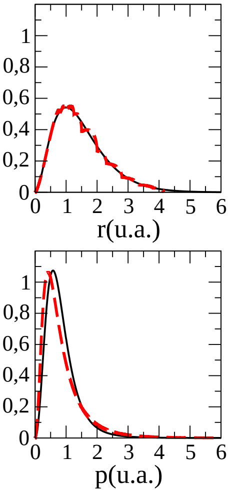

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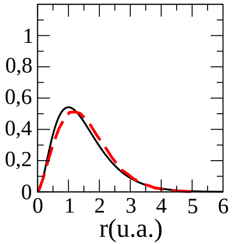
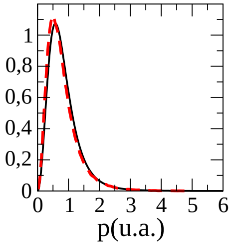
- Continous Distributions

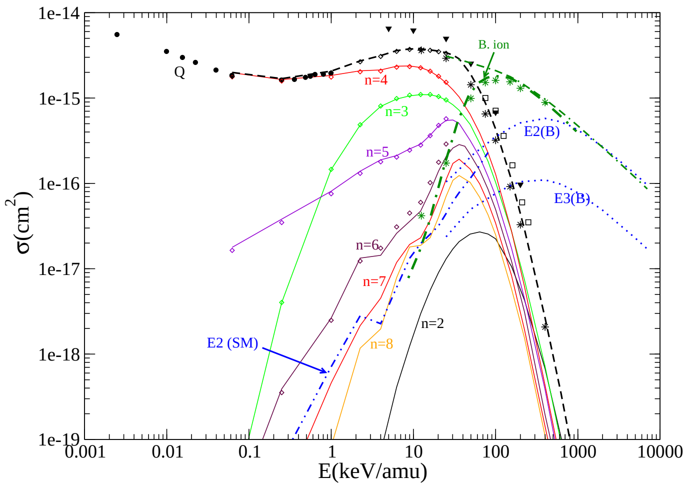
  Distributions: Gaussian, Rackovic, Cohen, Eichenauer, etc.

$$\rho(E) = K_1 e^{-K_2\left(\frac{Z_H}{\sqrt{-2E}} - 1.2\right)^2}$$





B^{5+} +H(1s) Cross Sections



L.F. Errea, F. Guzmán *et al.* PPCF **48** 1585(2006)

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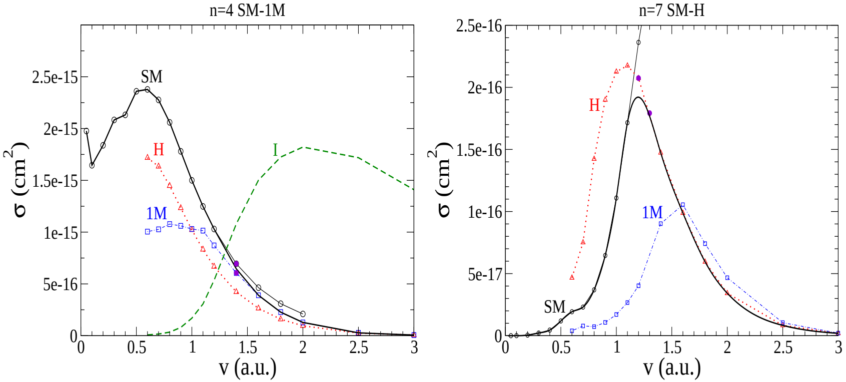
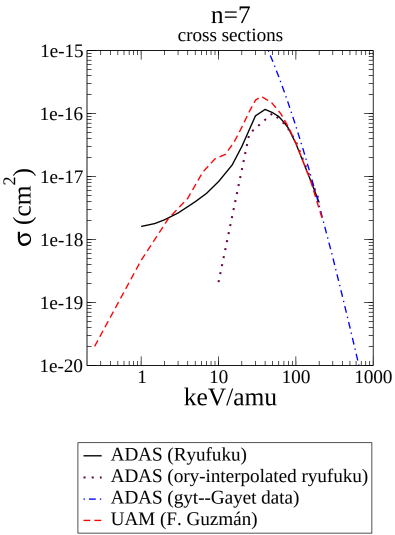
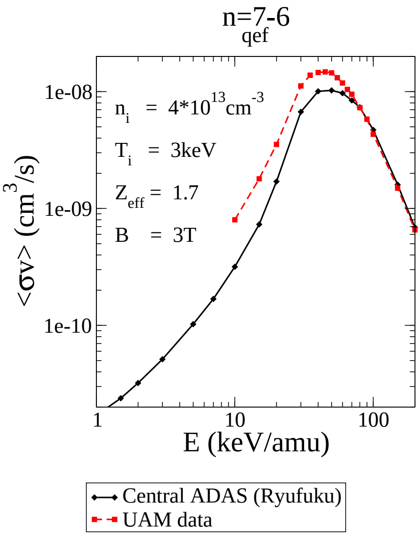


ADAS comparison

B^{5+} + H



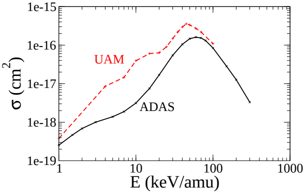


ADAS comparison

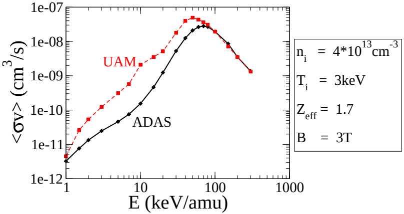
Ne^{10+} + H and Ar^{18+} + H

Ne^{10+} + H

n=11

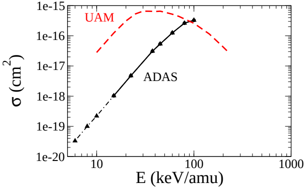


n=11-10 λ=524.92 nm

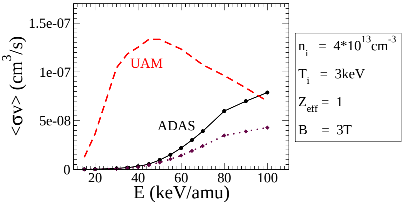


Ar^{18+} + H

n=16



n=16-15 λ=522.3nm

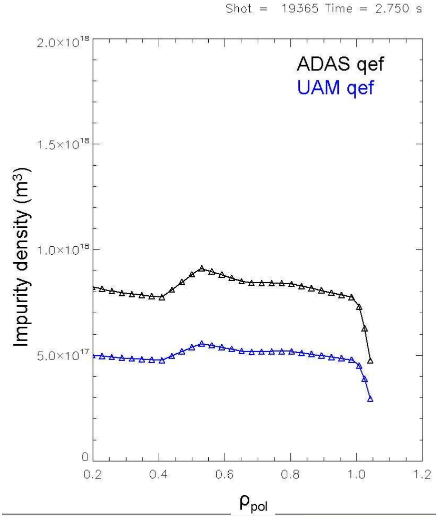


UAM: Errea *et al.* Nucl Inst. & Meth. Phys. Res. B **235**, 315 (2005)

CXRS Densities from QEF

CHEAP Results from ASDEX-U

Shot 19365; BV profile (7-6)



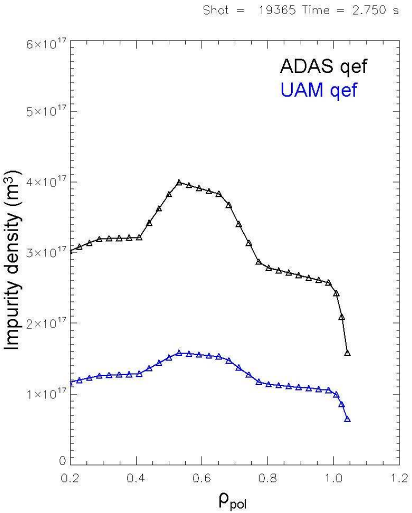
Radial density profile obtained from the fitting of calculated intensity for the transition to the experimental one using the CHEAP code in different shots in ASDEX-U.

- Black: Using ADAS qef data set
- Blue: Using UAM qef data set

CXRS Densities from QEF

CHEAP Results from ASDEX-U

Shot 19365; NeX profile (11-10)



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Conclusions

- A wide range of energy cross sections is achieved by overlapping different methods in its adequate energy.
- Adequate resembling of quantal initial conditions in each situation is needed for CTMC calculations.
- Cross sections accuracy is fundamental to obtain impurities densities by CXRS. There are big differences between the different calculations in cross sections.
- Experimental methods which help in providing recommended cross sections are needed.

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